The phase diagrams of quantum van der Waals liquids

Dam Thanh Son (University of Chicago)

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Ref.: D.T. Son, M. Stephanov, H.-U. Yee, 2006.01156

What is ultra-quantum mater?

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- Ultra-quantum matter: more quantum than quantum matter!

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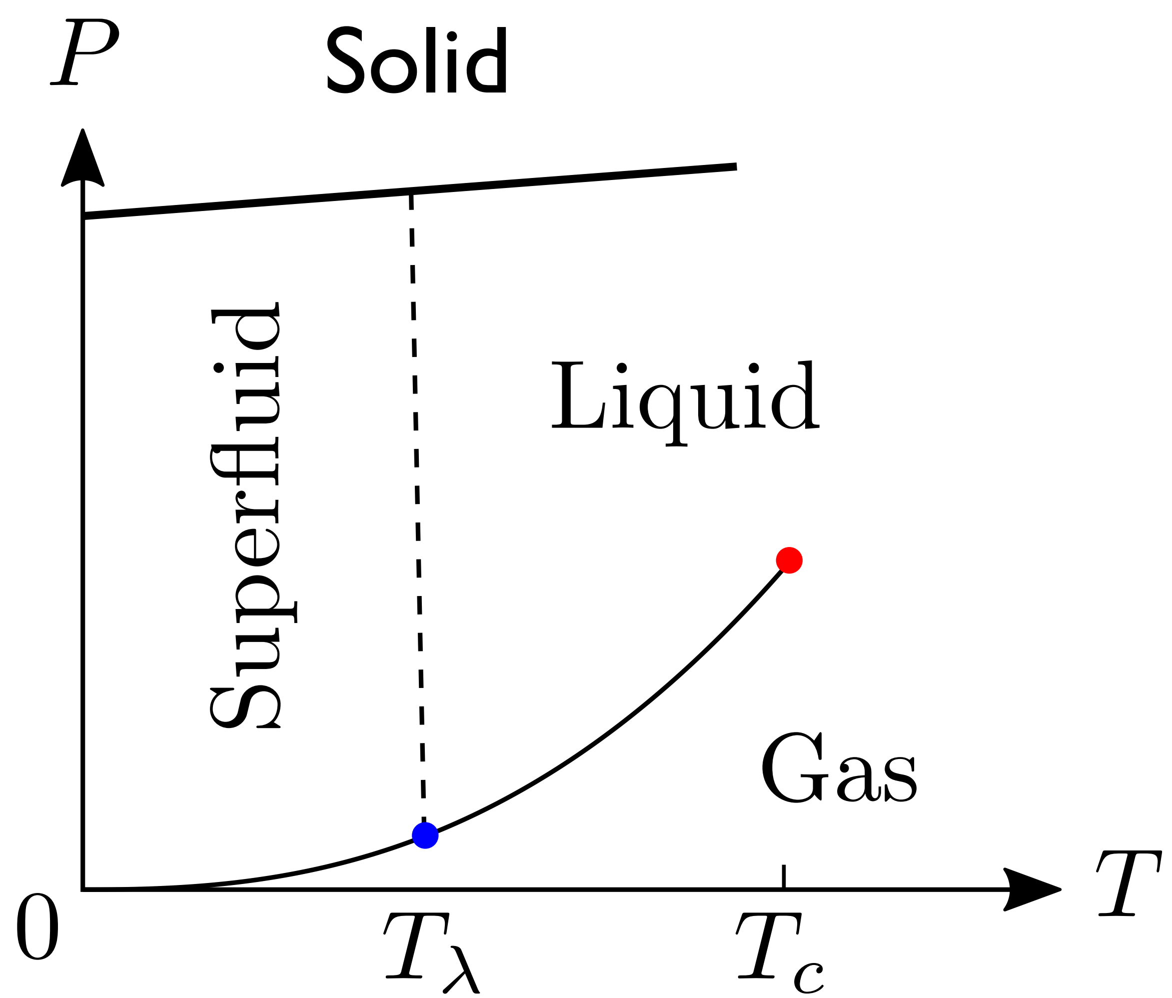
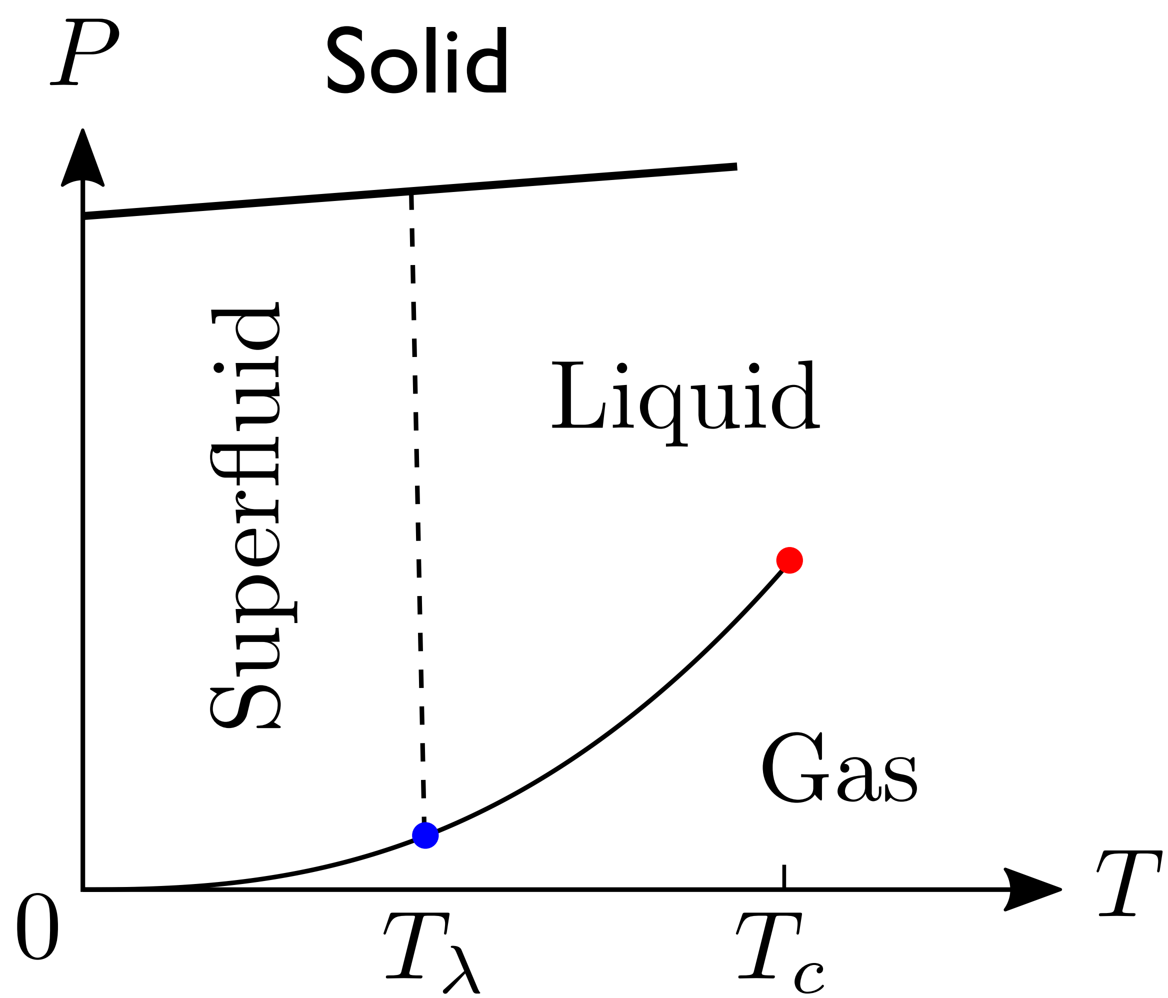
- Ultra-quantum matter: more quantum than quantum matter!
- Take a known quantum matter and make it more quantum

Phase diagram of helium

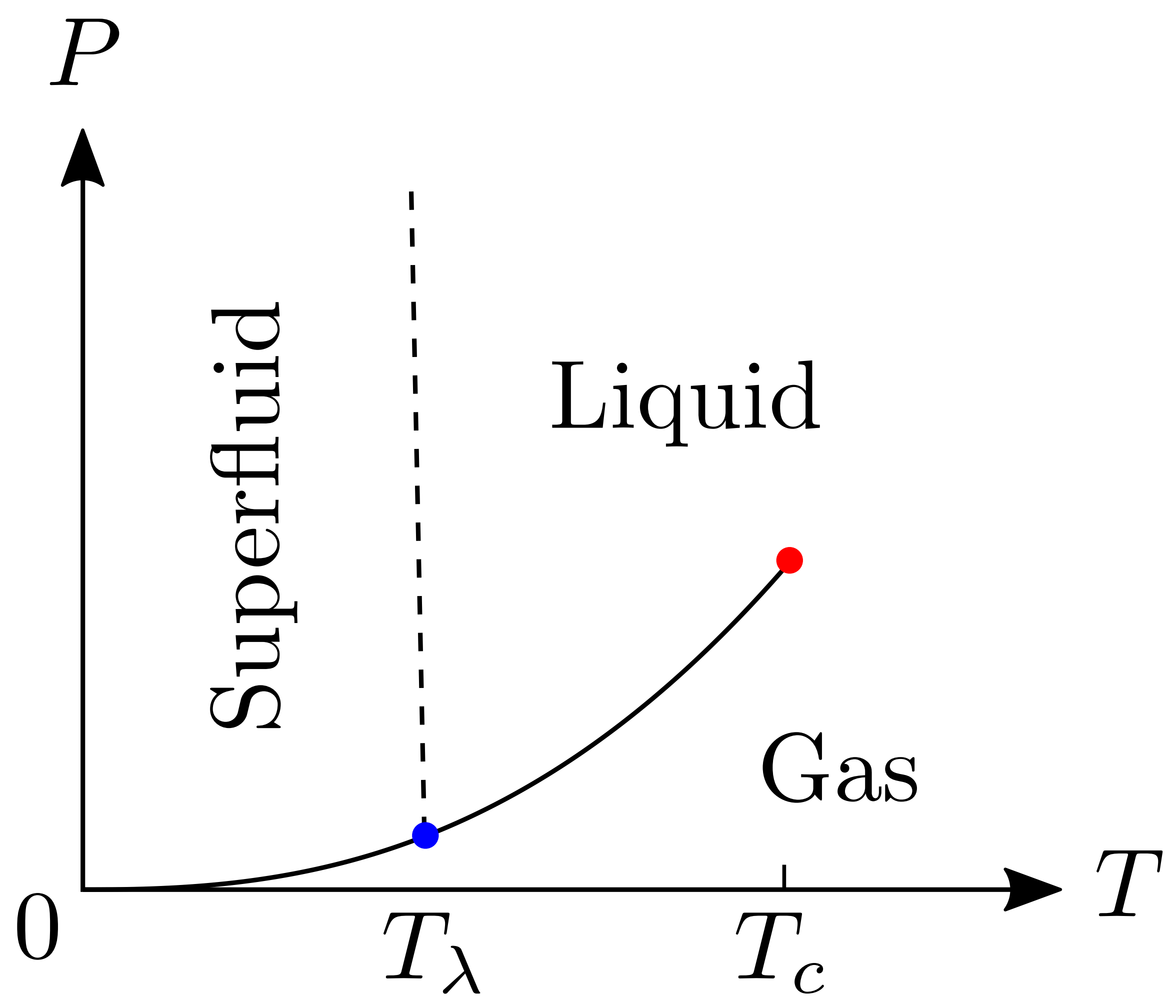


Phase diagram of helium

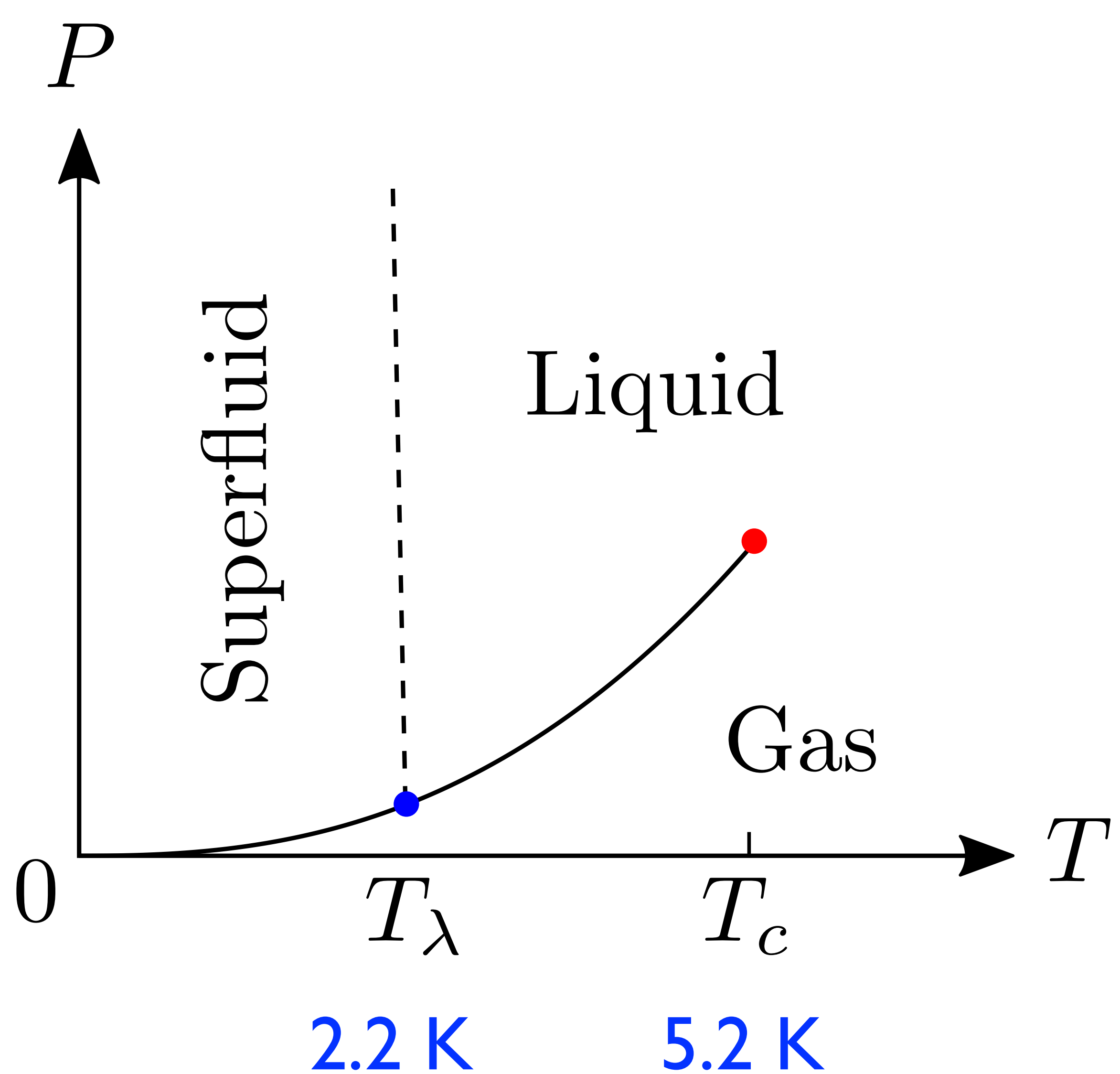
Charged condensate
(Gabadadze-Rosen)



Phase diagram of helium



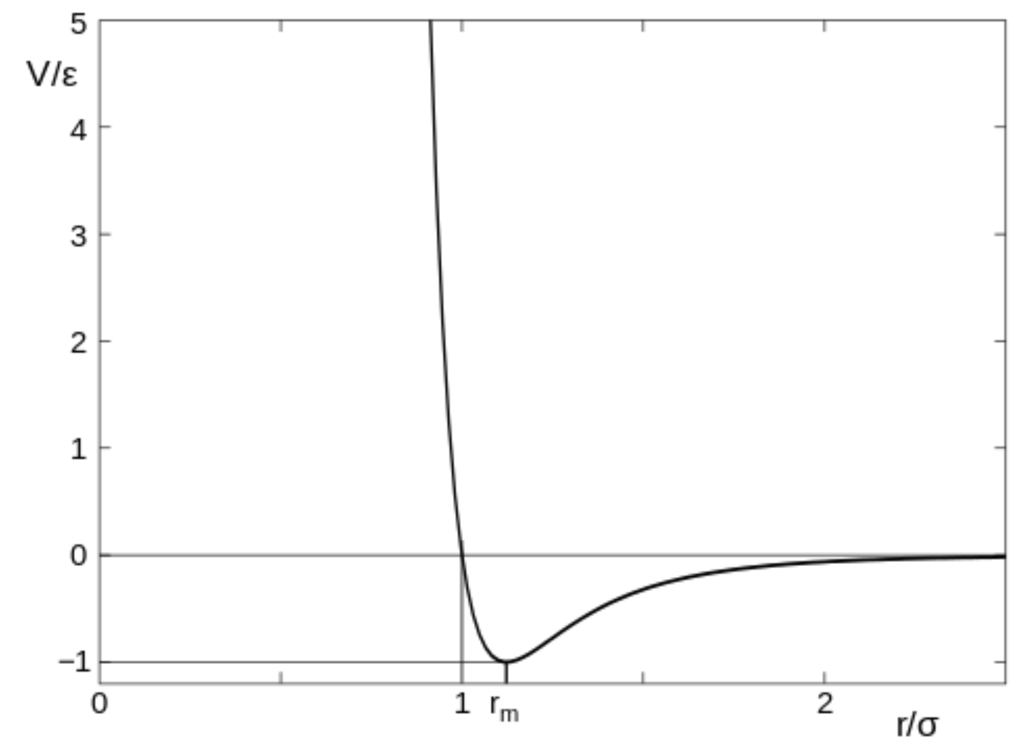
Phase diagram of helium



Quantifying quantum effects

- Lennard-Jones potential

$$V(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$



- de Boer parameter

$$\Lambda = \frac{\hbar}{\sigma\sqrt{m\epsilon}}$$

- $\Lambda \ll 1$: classical liquid/solid (zero point fluctuations small)

Table 21.13 Parameters of the Lennard–Jones Potential

Substance	σ (Å)	ϵ/k_B (K)
He	2.556	10.22
H_2	2.928	37.00
D_2	2.928	37.00
Ne	2.749	35.60
Ar	3.405	119.8
Kr	3.60	171
Xe	4.100	221
O_2	3.58	117.5
CO	3.763	100.2
N_2	3.698	95.05
CH_4	3.817	148.2

$\Lambda = 0.436$

$\Lambda = 0.275$

Berry, Rice, Ross, *Physical Chemistry*

“Ultra-quantum helium”

- Imagine that one lowers the mass of the helium nucleus, keeping it bosonic. What would happen?
- Born-Oppenheimer approximation: potential between atoms unchanged
- lower m, larger Λ (more quantum)
- Thus lighter helium will not solidify at zero temperature and zero pressure, but what will happen to the phase diagram?

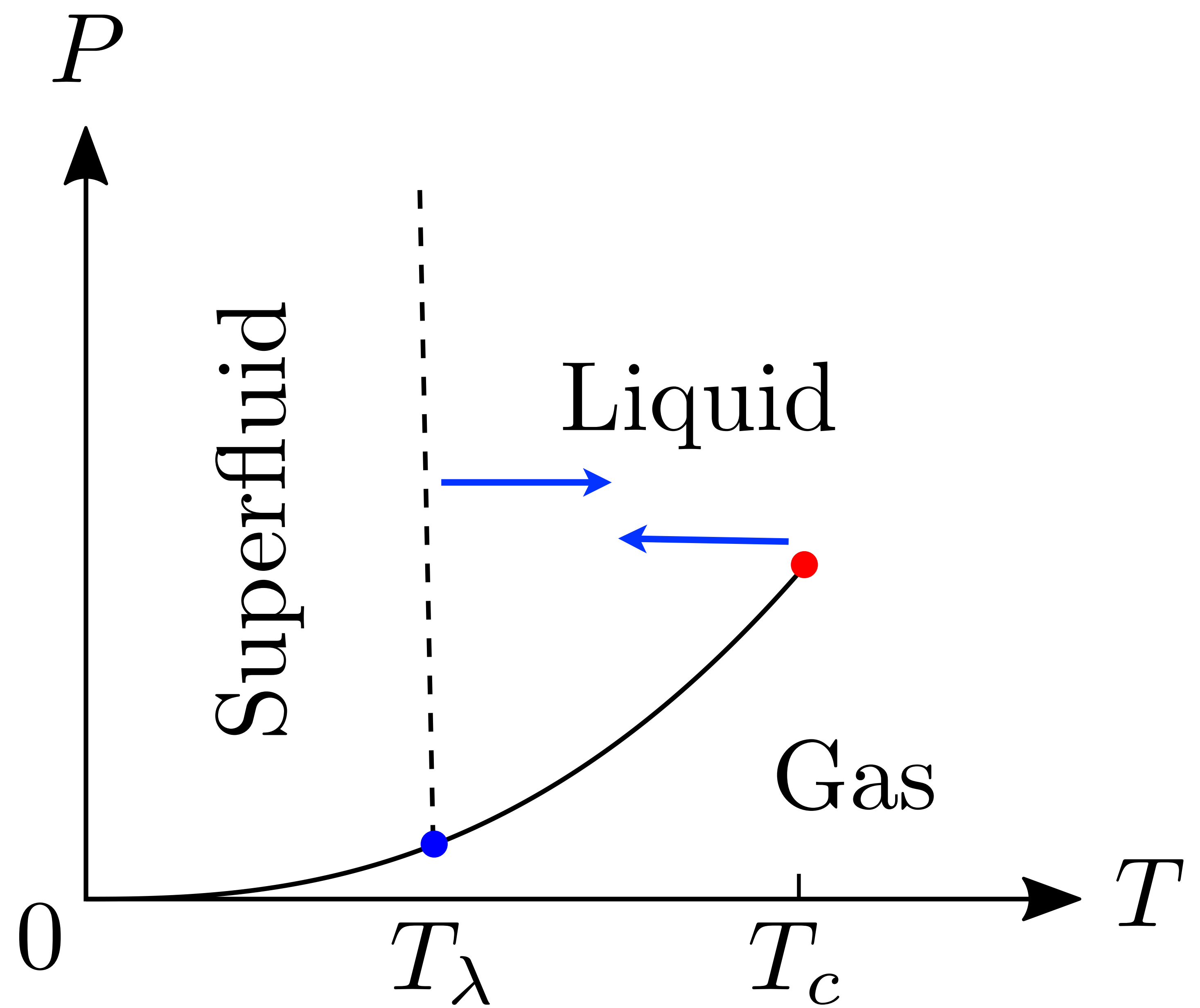
Superfluid phase transition

- Naively: superfluid phase transition = BEC transition

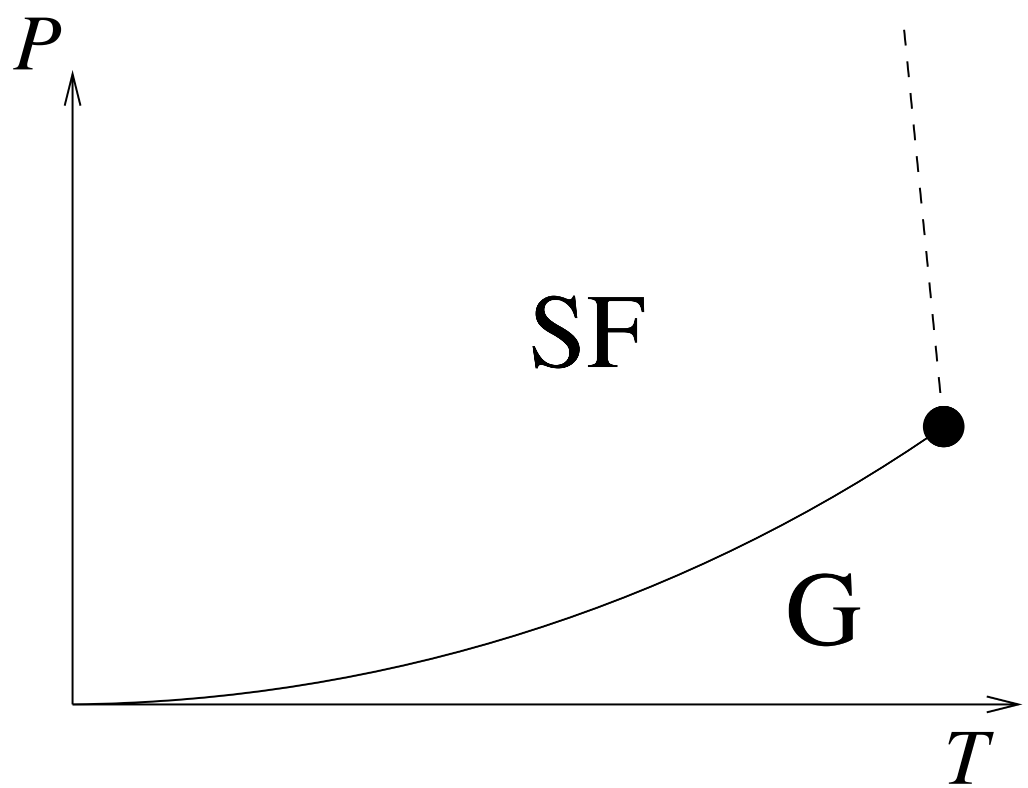
$$T_c = \# \frac{\hbar^2 n^{2/3}}{m}$$

- T_c increases when the mass decreases
- Critical temperature of liquid-gas phase transition decreases with nuclear mass (neon)

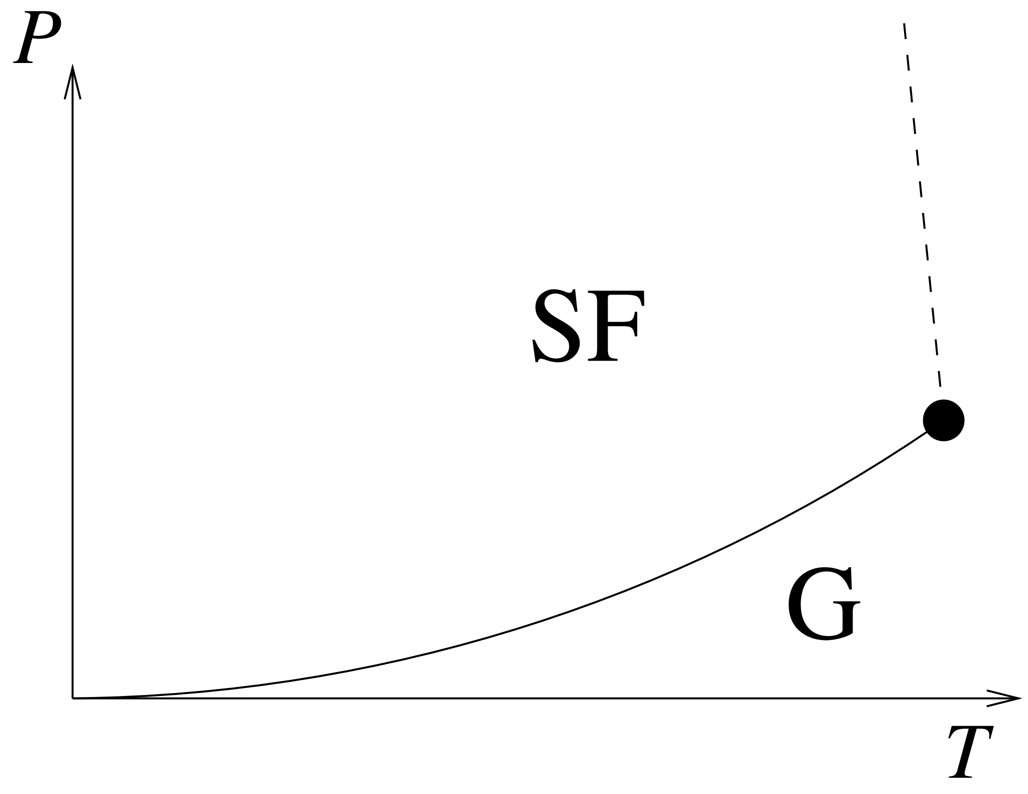
What happens when two temperatures coincide?



Multicritical point?



Multicritical point?



IMPOSSIBLE

Theory of the multicritical point

$$\Omega(\psi,\phi) = \frac{t}{2}\phi^2 + \frac{u}{4}\phi^4 - h\phi$$

Z2 liquid-gas critical point

$$+(t+\tilde{m})|\psi|^2 + \frac{\lambda}{2}|\psi|^4$$

U(1) superfluid phase transition

$$-\alpha\phi|\psi|^2 - \gamma\phi^2|\psi|^2$$

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But we can tune only 3 parameters: P, T, m

Mean-field model

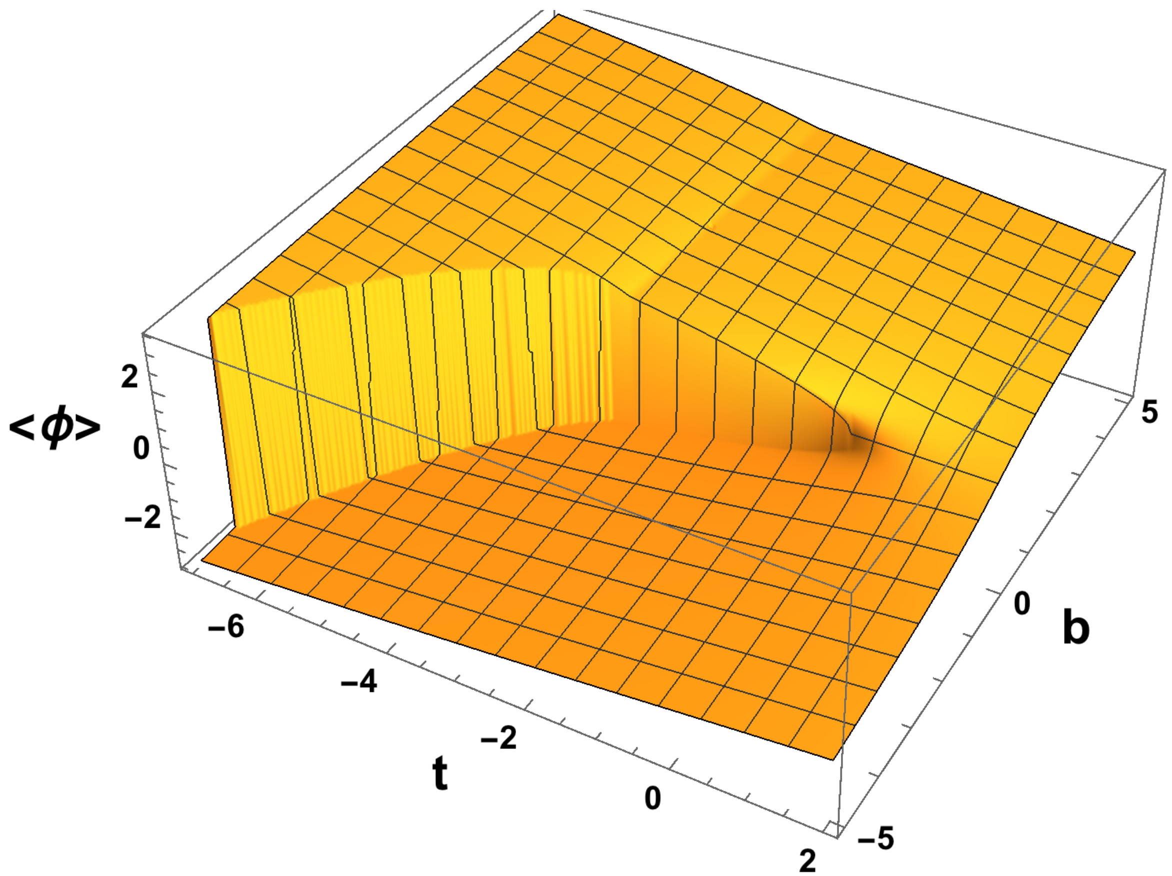
- To get ideas of what can happen, we investigate numerically the mean field model

$$\Omega(\phi, |\psi|) = \frac{t}{2}\phi^2 + \frac{u}{4}\phi^4 - h\phi + (t + \widetilde{m})|\psi|^2 + \frac{\lambda}{2}|\psi|^4 - \phi|\psi|^2$$

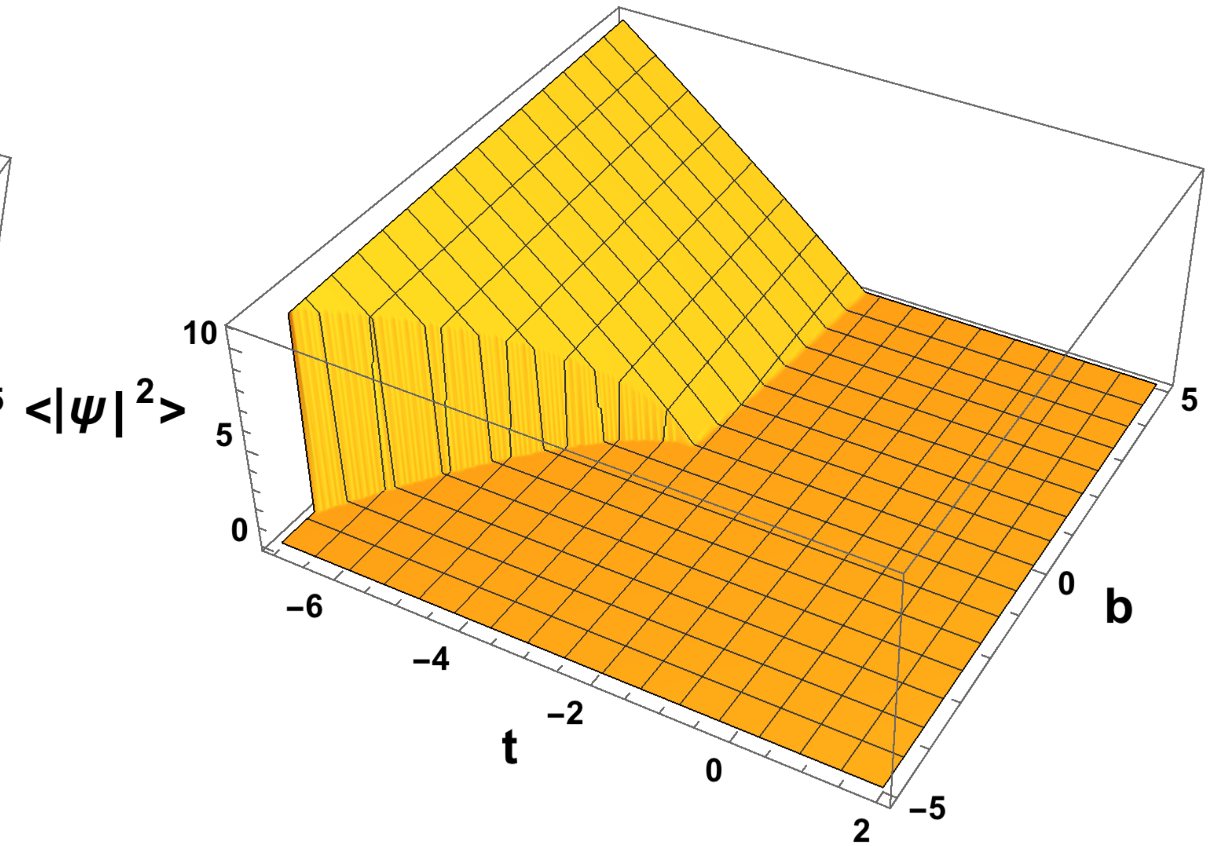
$\tilde{m}$: mass of the atom
t: temperature
h: pressure

In the numerics we fix $u = 1$, $\lambda = 1/2$

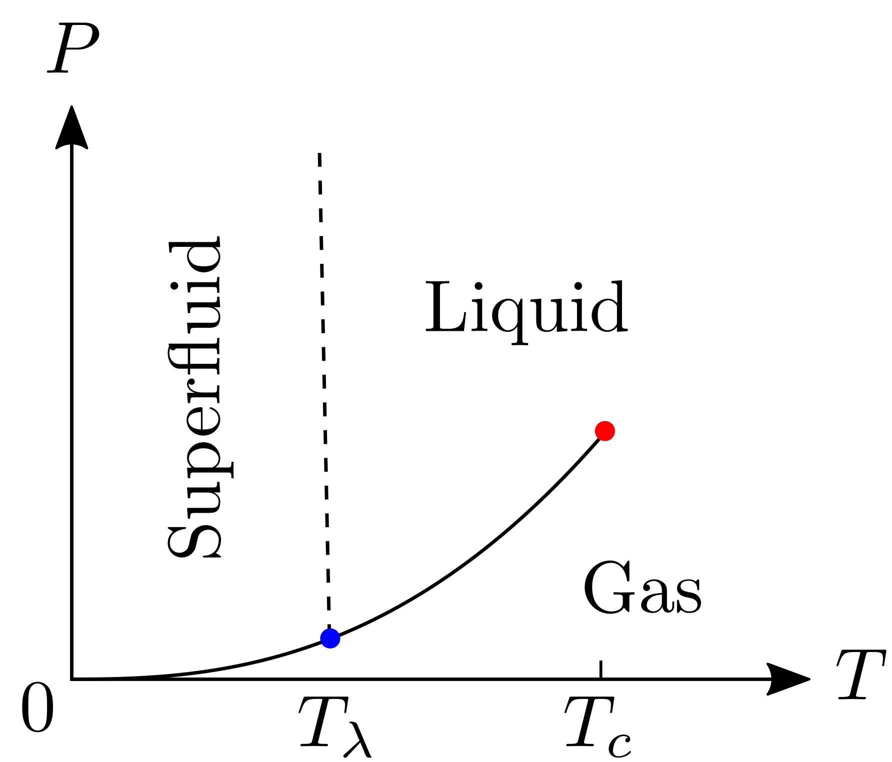
$$\tilde{m} = 5$$



<φ>
2
0
−2
−6
−4
−2
0
2
t
5
0
−5
b

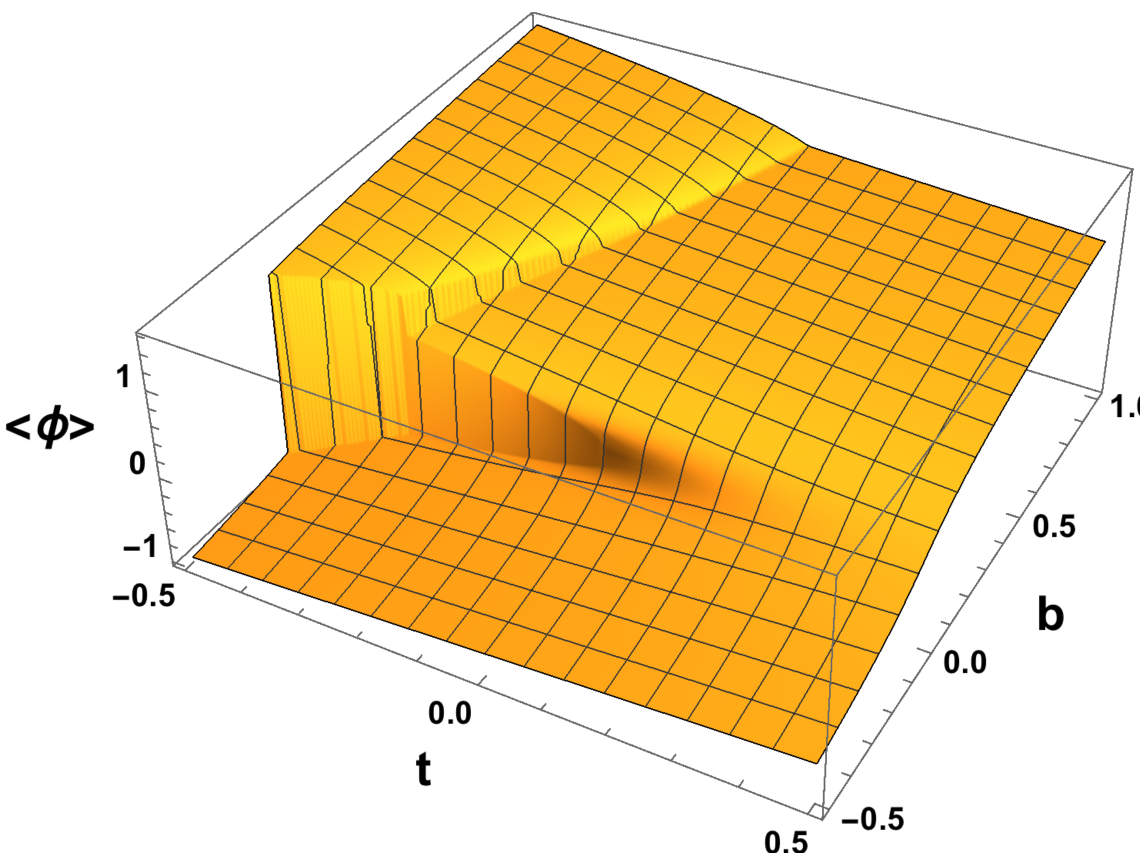


<|ψ|²>
10
5
0
−6
−4
−2
0
2
t
5
0
−5
b

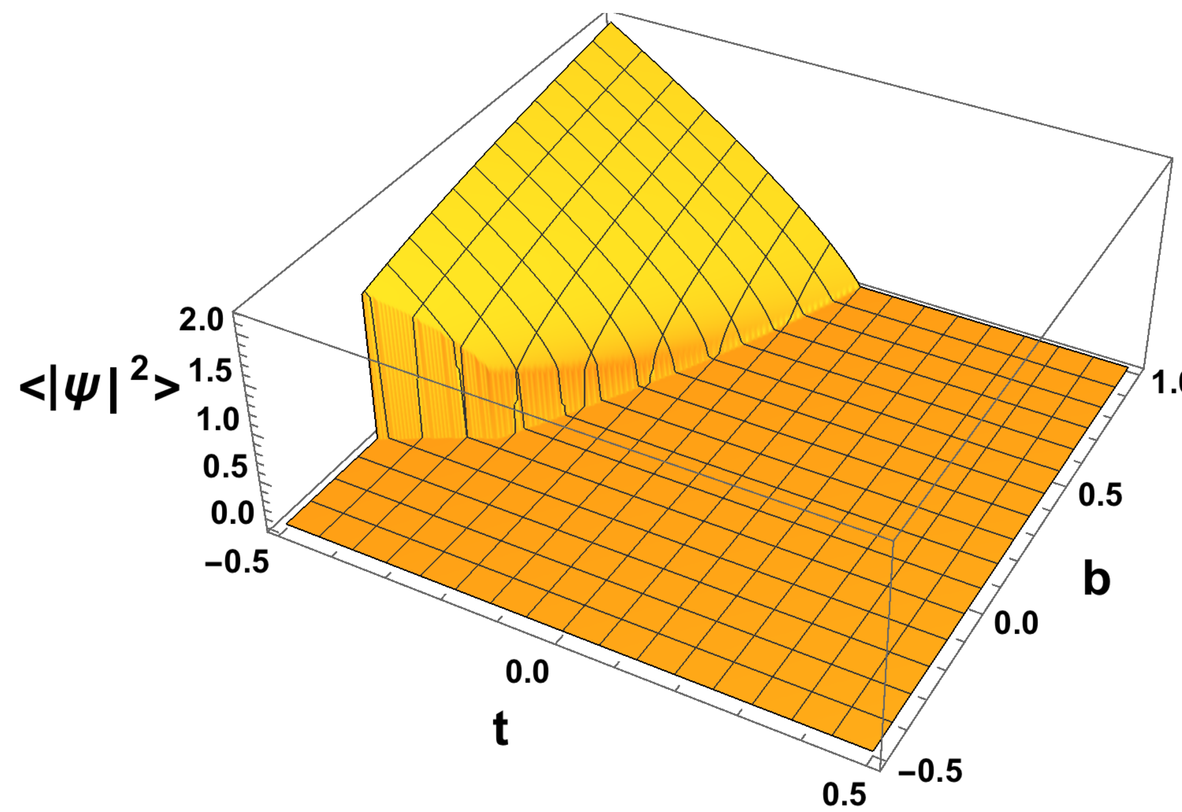


P
T
0
Superfluid
Liquid
Gas
T_λ
T_c

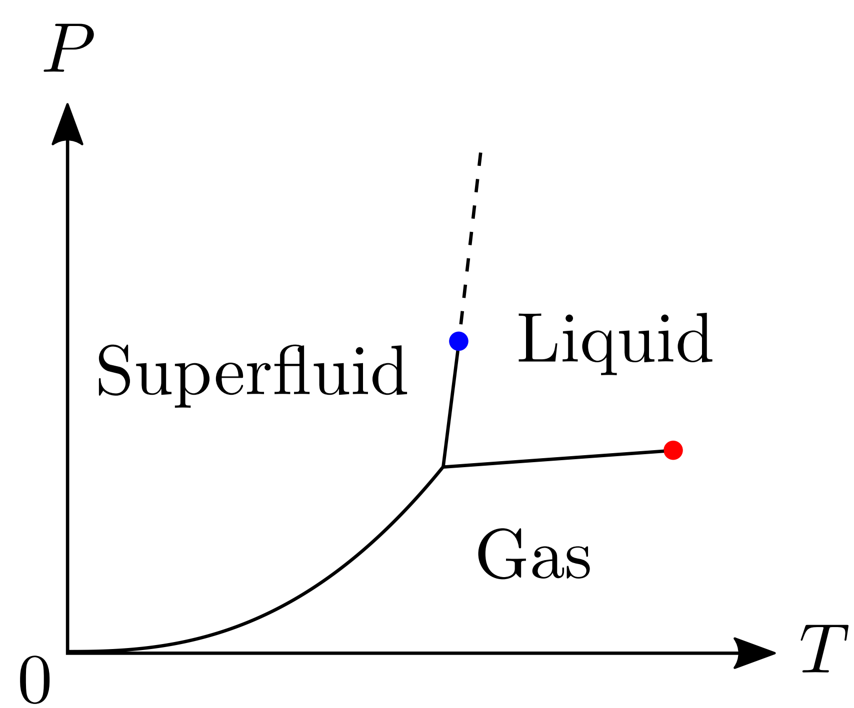
$$\tilde{m} = 1$$



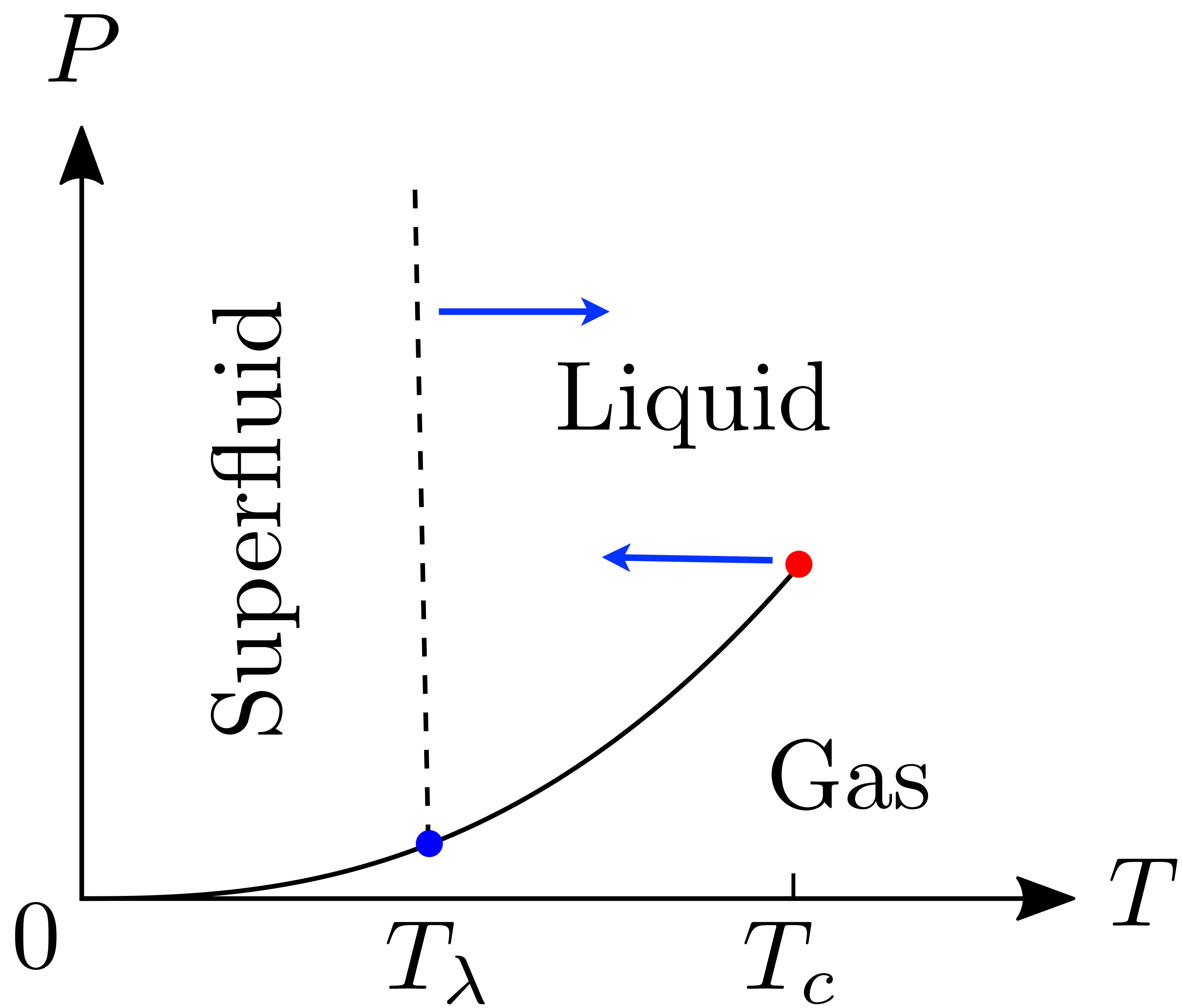
<φ>
1
0
−1
−0.5
0.0
t
0.5
−0.5
0.0
0.5
b
1.0



<|ψ|²>
2.0
1.5
1.0
0.5
0.0
−0.5
0.0
t
0.5
−0.5
0.0
0.5
b
1.0



P
Superfluid
Liquid
Gas
0
T


P
Superfluid
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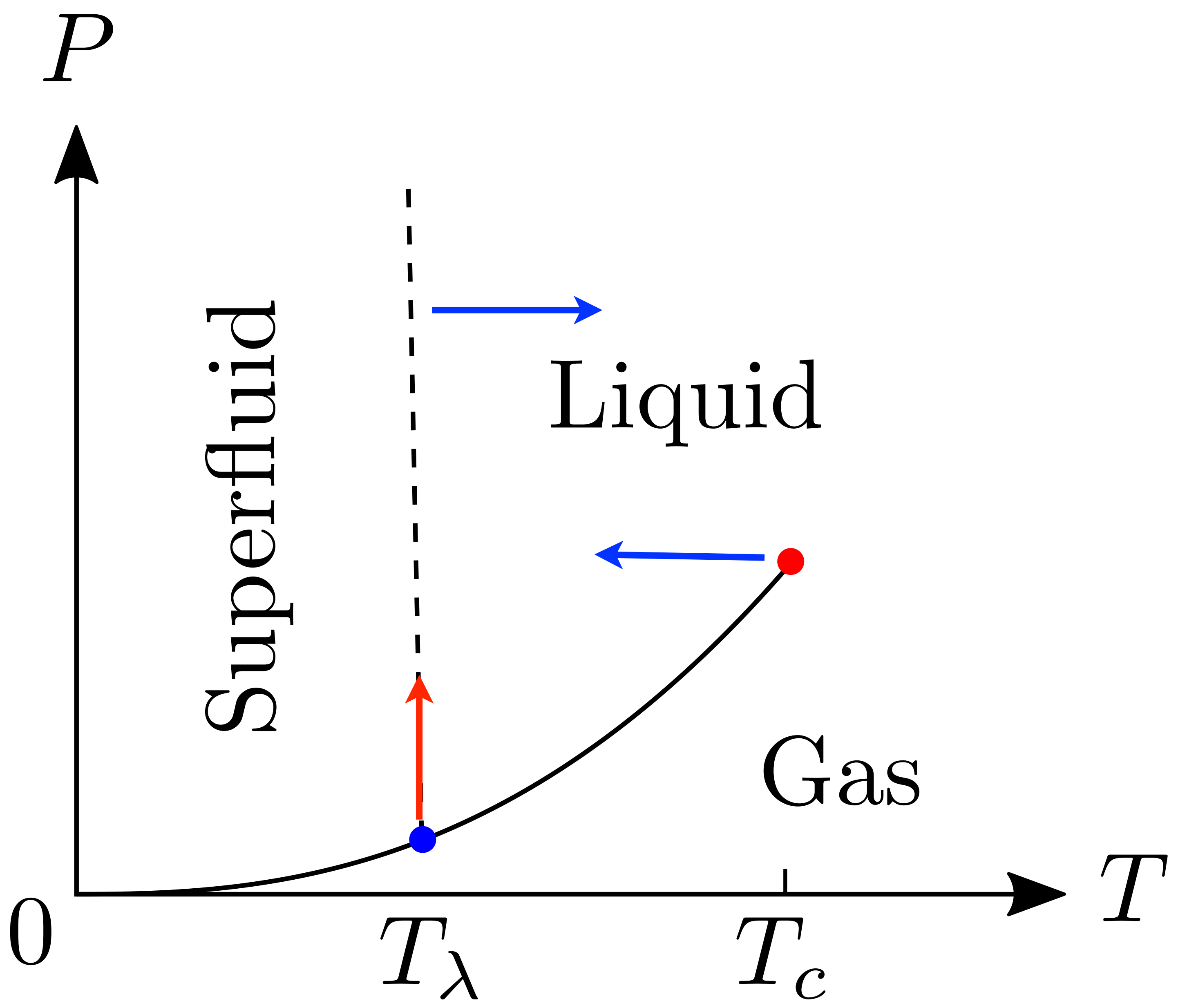
T
0
T_λ
T_c



P
Superfluid
Liquid
Gas
T
0
T_λ
T_c

Induced first-order superfluid phase transition

$$V(\phi,\psi) = \cdots + \frac{1}{2\chi}(\delta\phi)^2 - \delta\phi|\psi|^2 + \frac{\lambda}{2}|\psi|^4$$

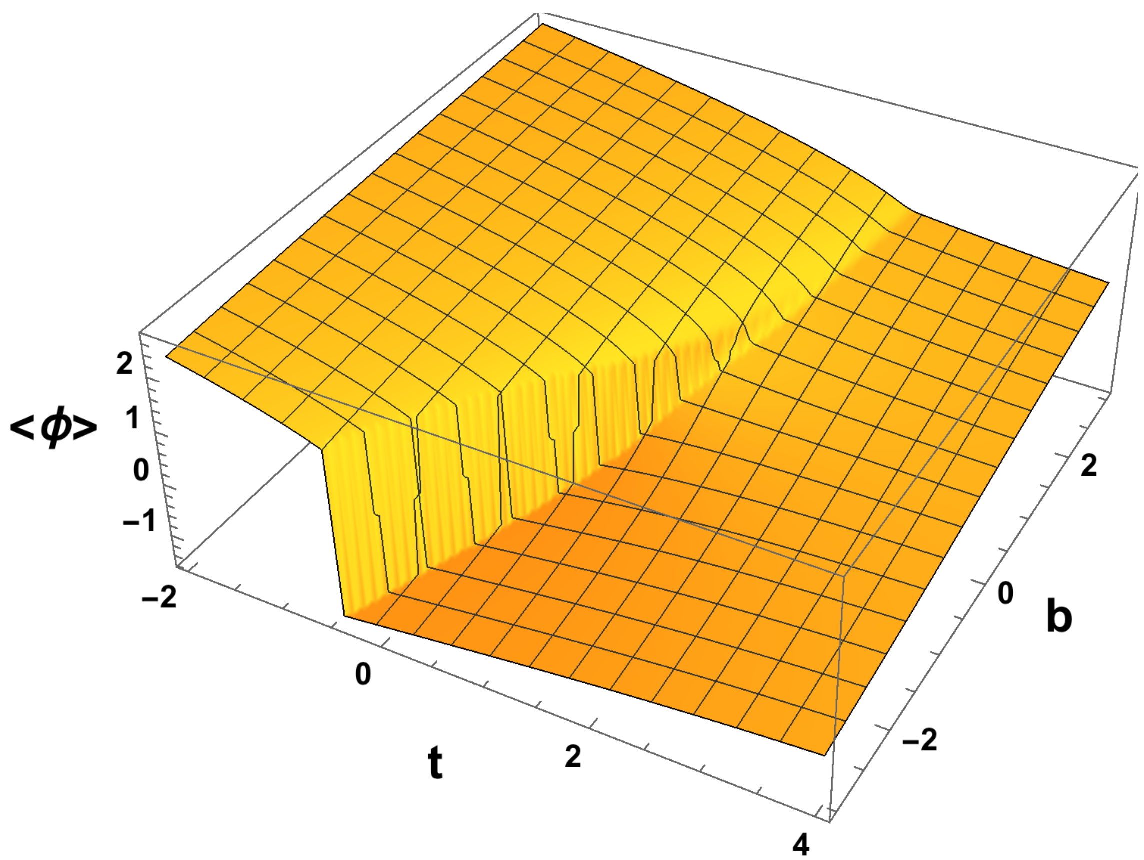
Integrate out fluctuations of liquid-gas order parameter

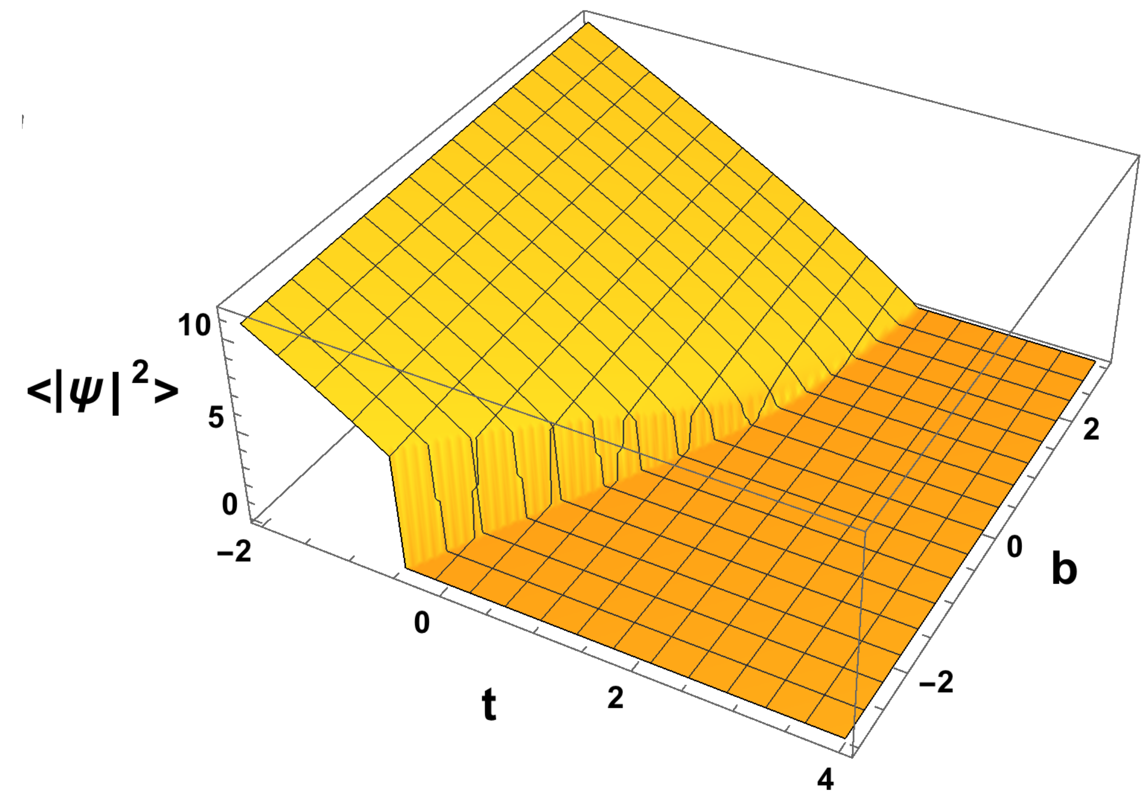
$$V_{\text{eff}}(\psi) = \cdots + \left(\frac{\lambda}{2} - \frac{\chi}{2}\right)|\psi|^4$$

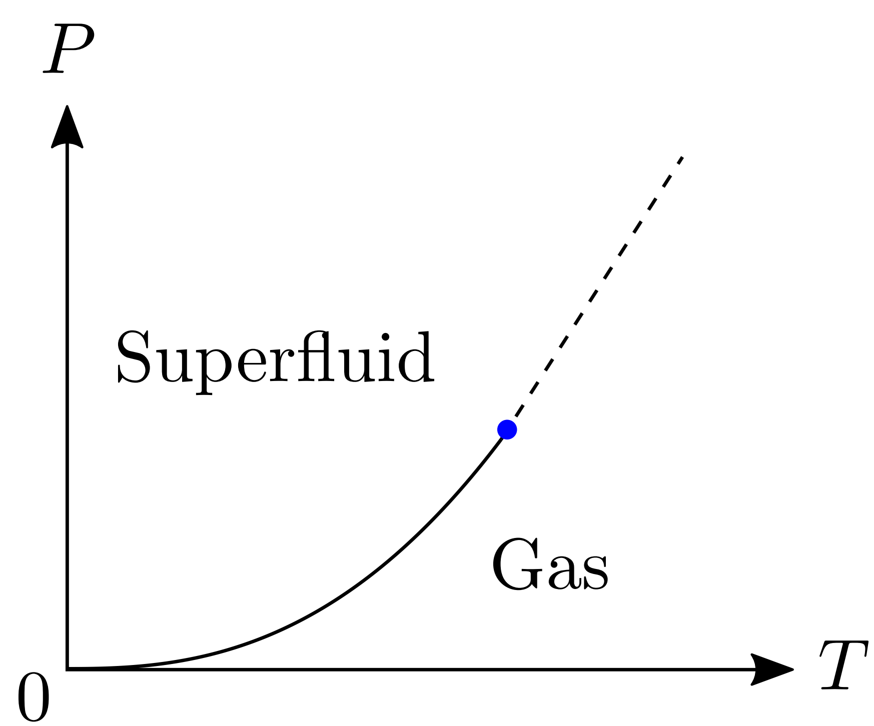
Near the liquid-gas critical point $\chi \rightarrow \infty$, and the $|\psi|^4$ coefficient becomes negative

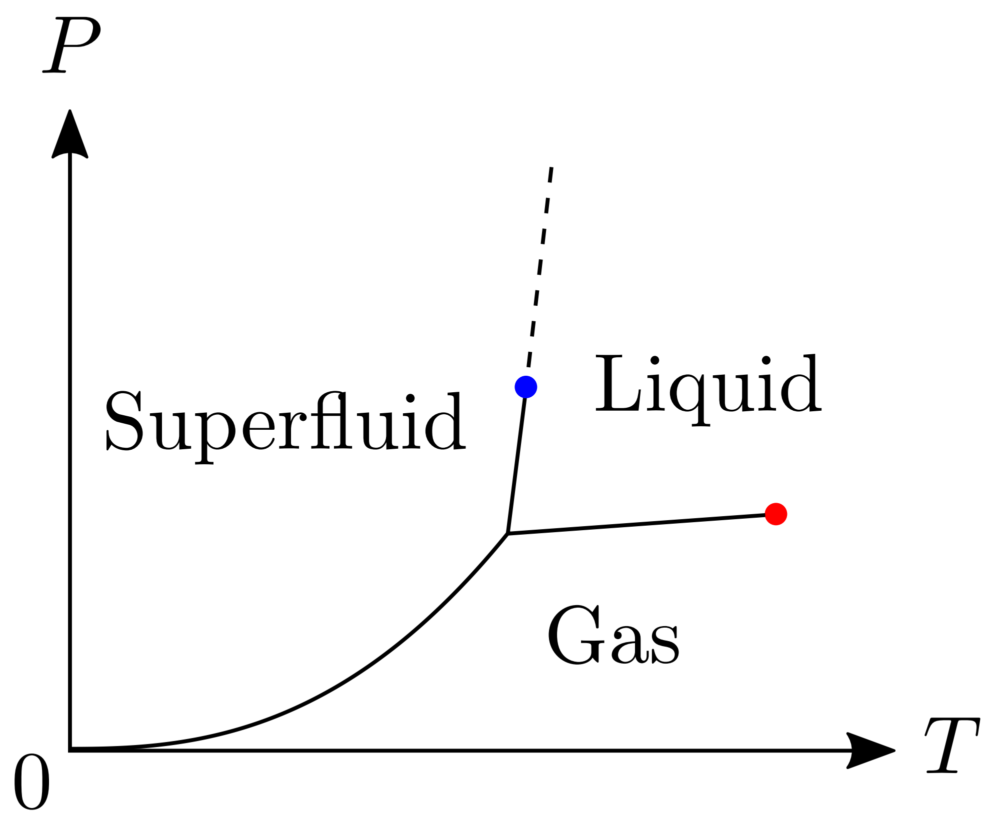
The superfluid phase transition becomes first order

$$\tilde{m} = -1$$

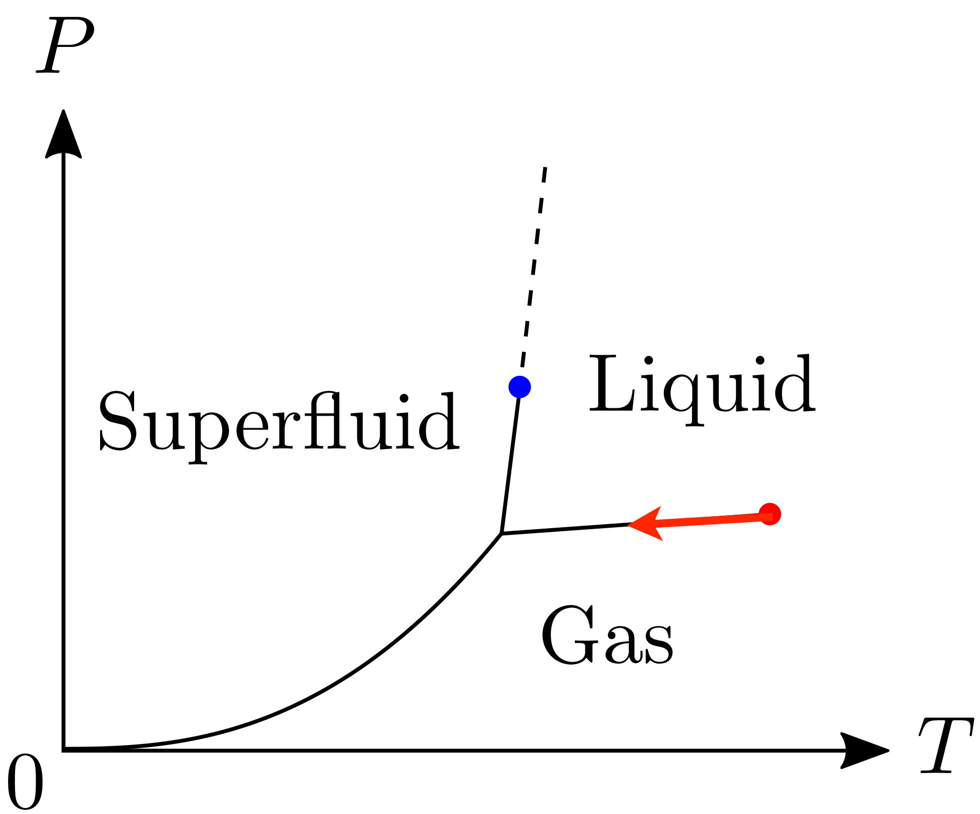








P
Superfluid
Liquid
Gas
0
T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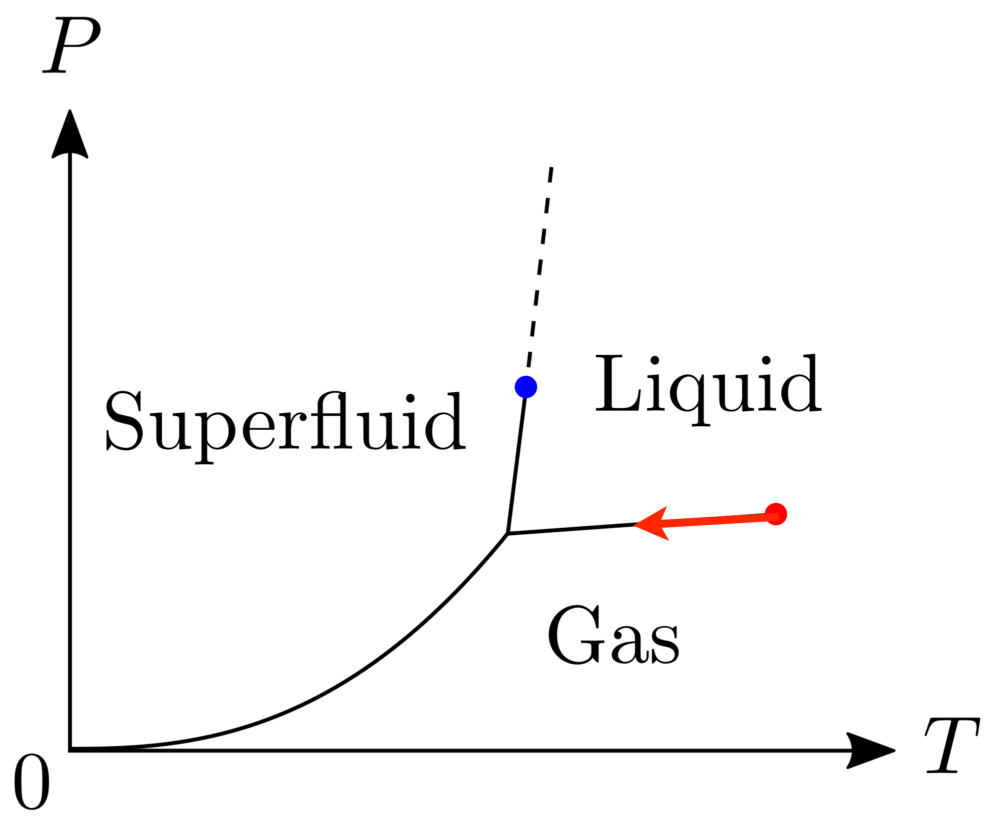
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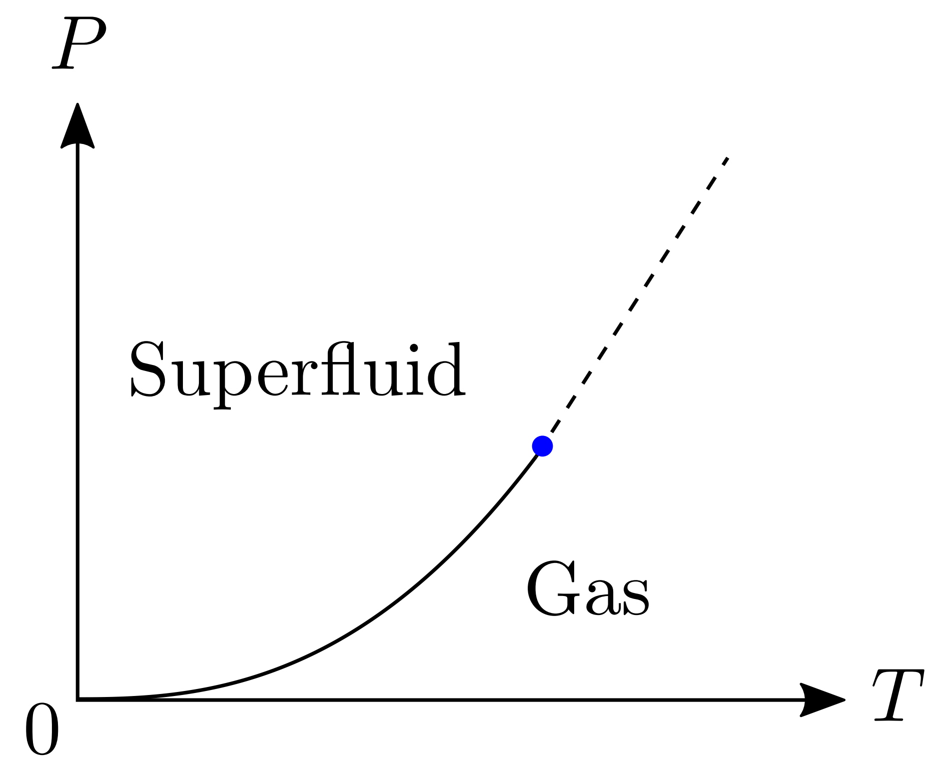
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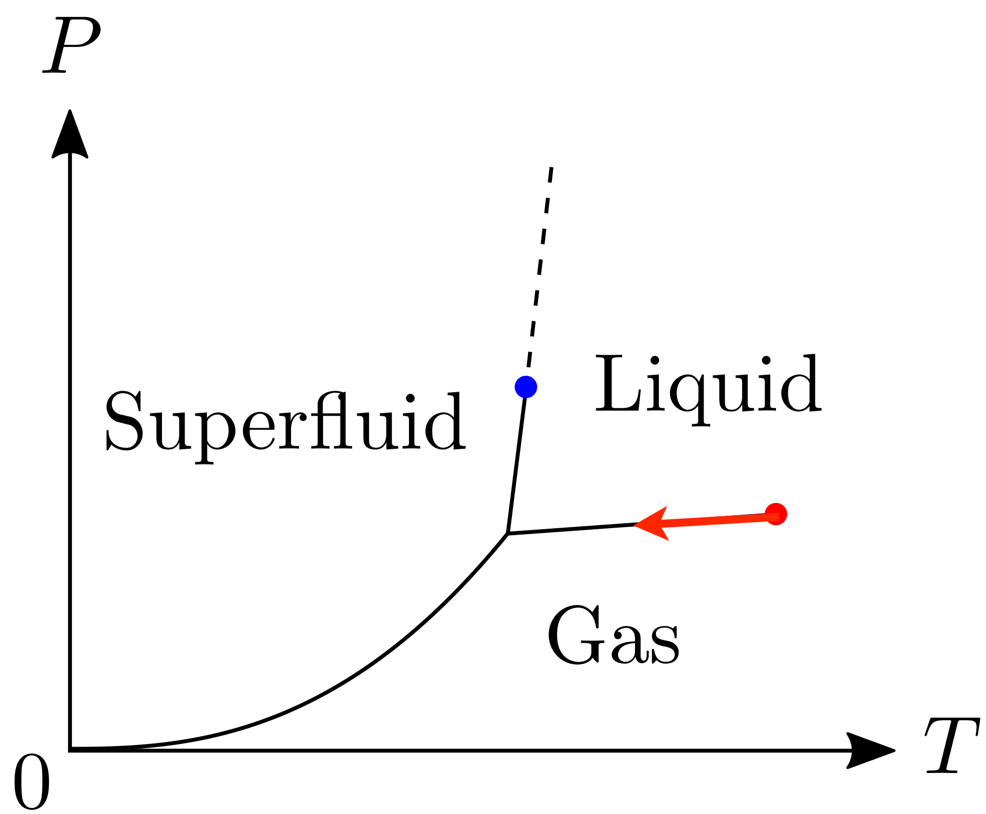
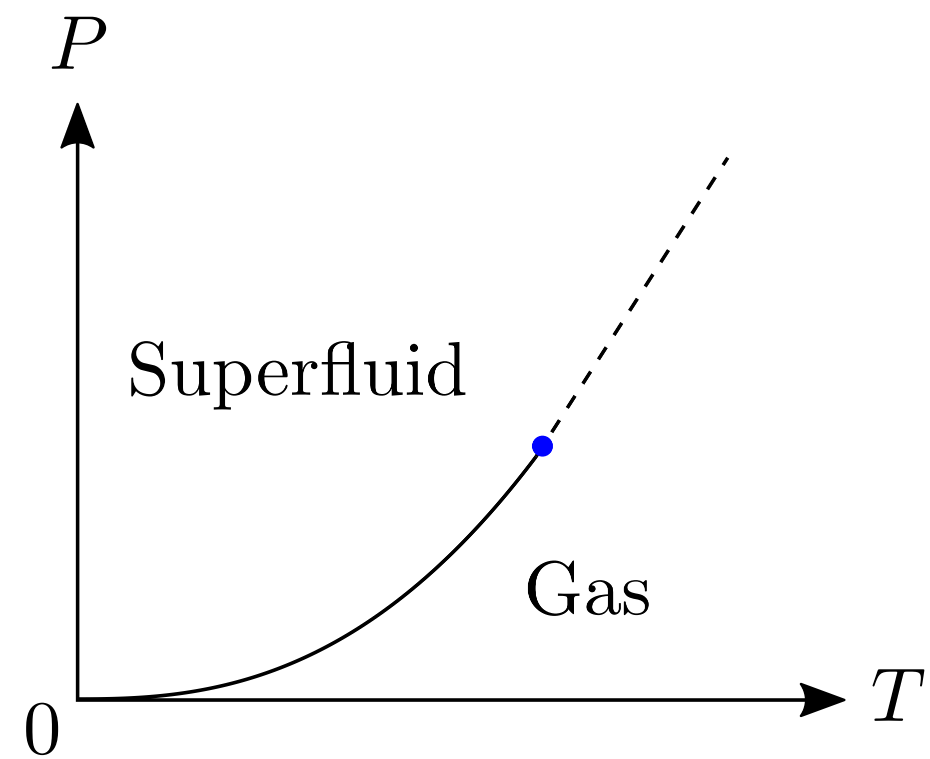
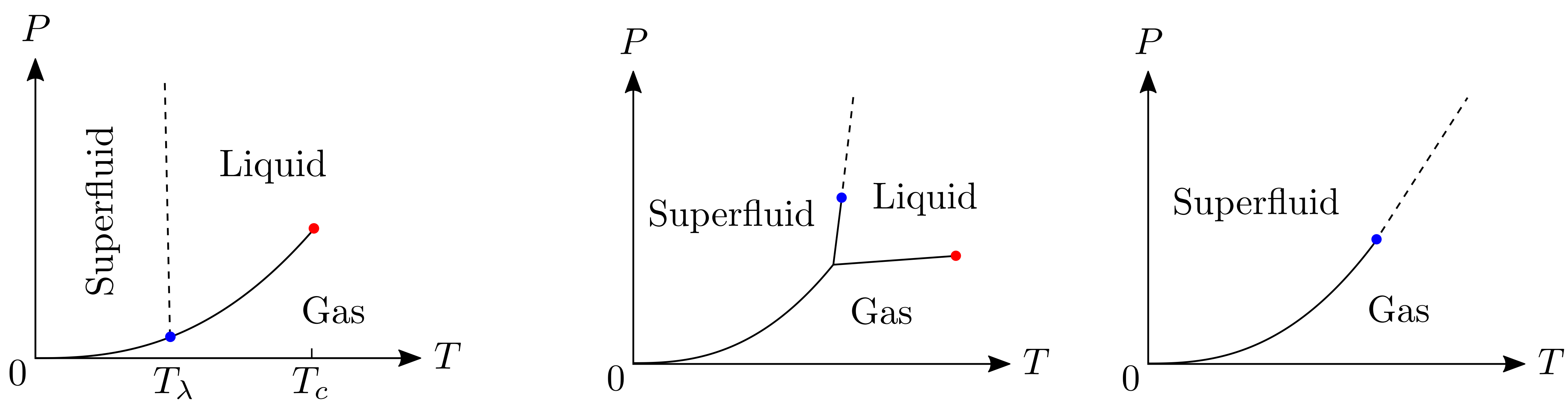




Liquid-gas critical point disappears under the superfluid phase transition line

Summary of mean field results

decreasing m



What is the range of m?

Unbinding phase transition

- Zwerger 2019: self-bound liquid disappears when the scattering length crosses zero
- The vicinity of this point can be reliably studied

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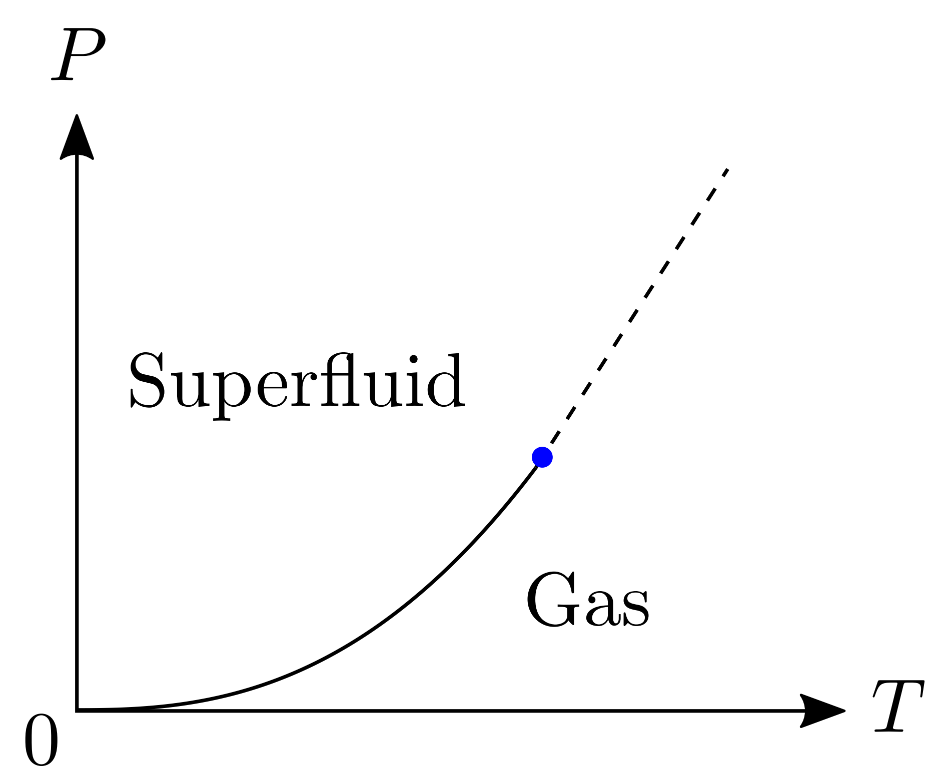

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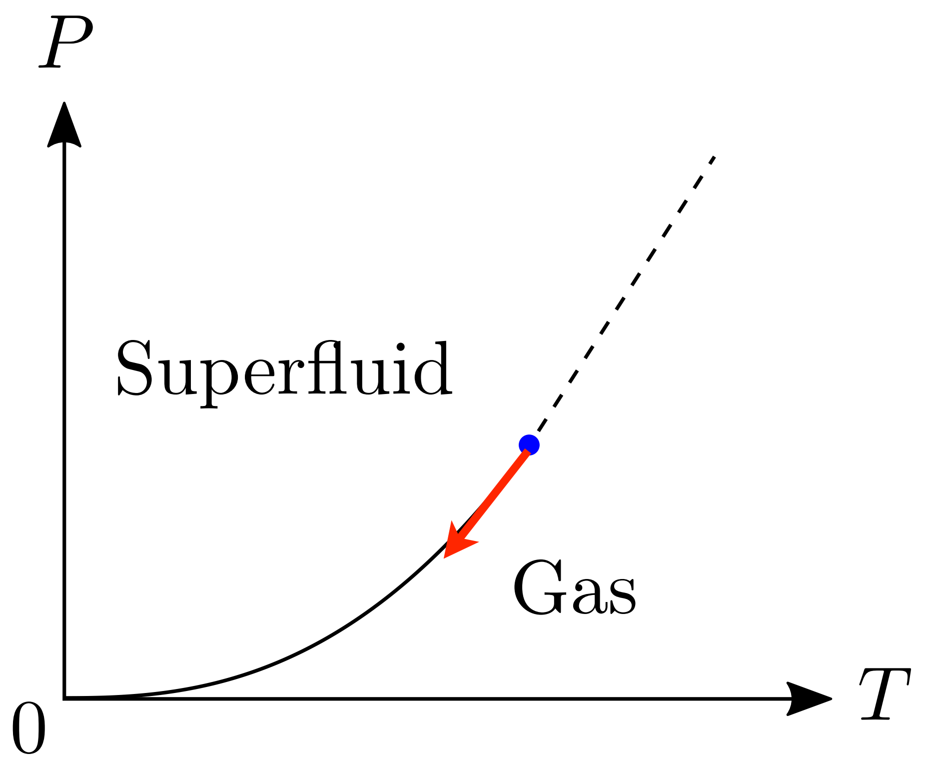


Dilute self-bound liquid

$$\frac{\Omega}{V} = \frac{g}{2}n^2 + \frac{G}{6}n^3 - \mu n,$$

$$g = \frac{4\pi\hbar^2 a}{m}$$

a: scattering length

a<0, *a*→0

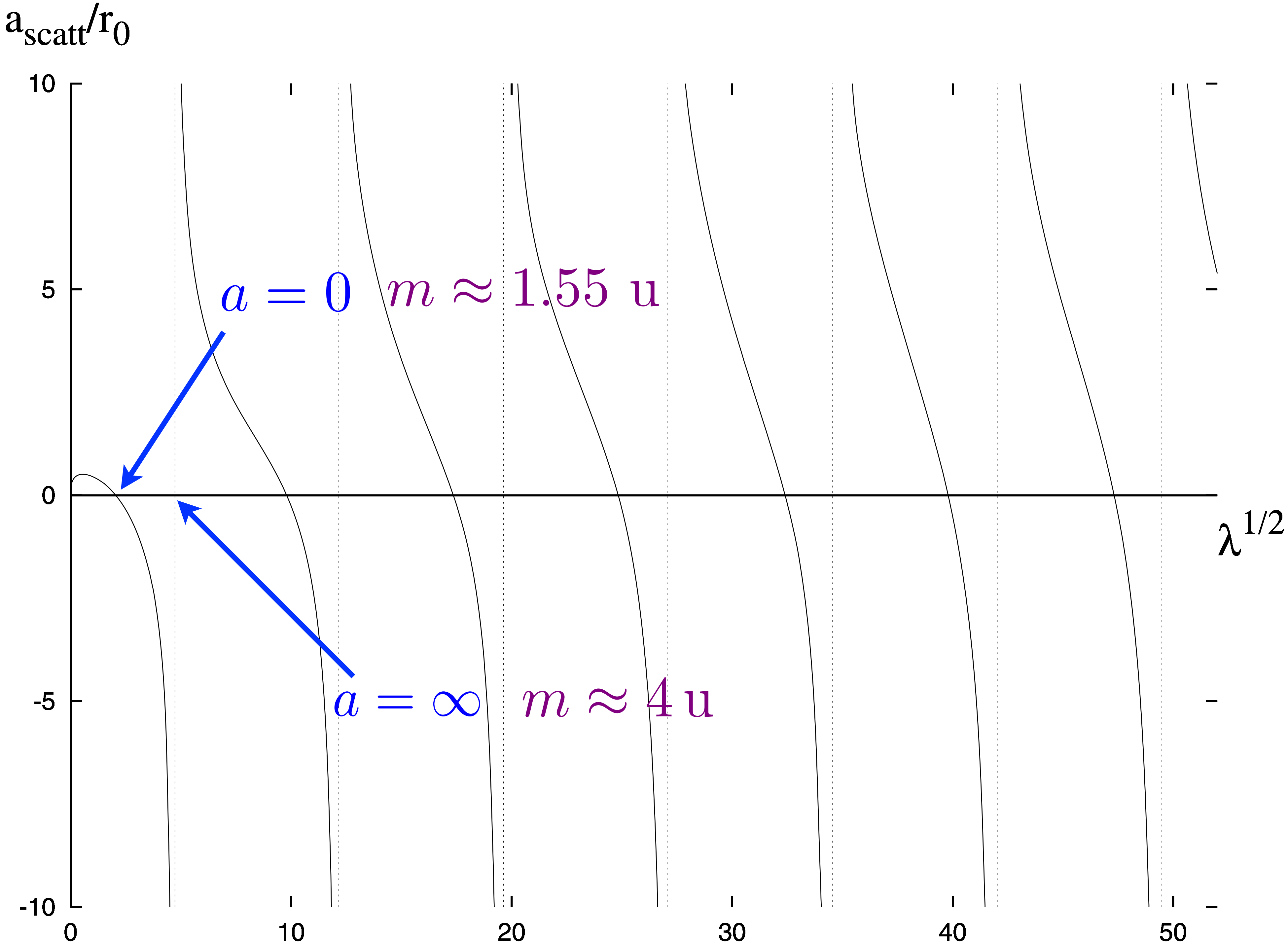
$$G = \frac{\hbar^2 D}{m}$$

D: three-body scattering hypervolume

(Shina Tan, 2008)

Reliable calculation in the regime $a^4 \ll D$, $D>0$

T=0: self-bound fluid with $n = \frac{3}{2}\frac{(-g)}{G} = 6\pi\frac{(-a)}{D}$



Gómez, Sesma, 2012

All transformations of the phase diagram happen between 4 u and 1.55 u

Physical realizations

- Unfortunately, helium does not have a bosonic isotope lighter than He-4
- (He-6 and He-8)

Muonic matter

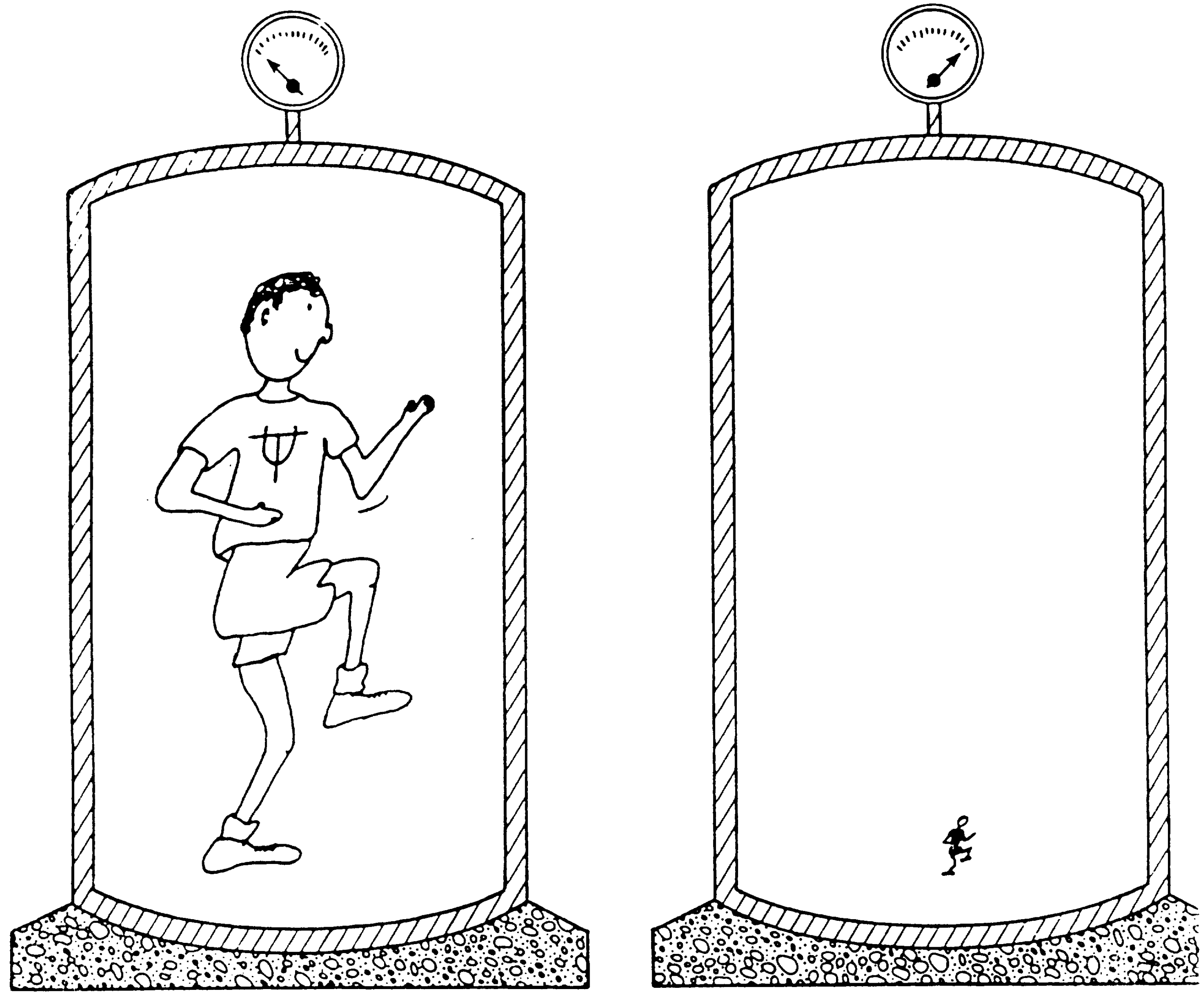


Figure 2. With sufficient increase of the neutrino pressure, all the electrons of all the atoms are converted to μ-mesons. The new all-μ-meson atoms are two-hundred times smaller than the original atoms.

J. Wheeler "Nanosecond matter" (1988)

Muonic quantum liquids

- "effective helium mass": the mass of the helium isotope with the same de Boer parameter
- replacing electrons by muons: reduce the effective helium mass by 207 times

	^{4}He	^{20}Ne	^{40}Ar	^{84}Kr	H_2	N_2	O_2	CO	CH_4
$M_{\rm eff}$	4	80.6	832	2790	9.50	545	722	595	517
$M_{\rm eff}(\mu)$	0.020	0.412	4.23	14.1	0.051	2.79	3.69	3.04	2.68

“Light hydrogen”

- Take hydrogen H_2 and replace protons by particles of change +1 with lesser mass
- muon: no self-bound liquid; pion or kaon: self-bound liquids.
- (lifetime 10^{-6} or 10^{-8} seconds, long compared to atomic scales)
- Biexcitons in solids?
  - want hole mass/ electron mass ~ 250-800

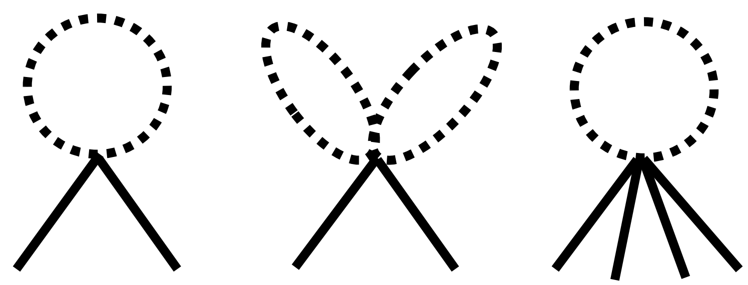
Conclusion

- Interesting possibilities for liquids more quantum than helium
- Can be simulated by quantum Monte-Carlo (Massimo Boninsegni, Youssef Kora, Shiwei Zhang)
- 2D systems?
- realizations? using cold atoms?

Extra slides

Finite temperature

- Integrate out nonzero Matsubara modes

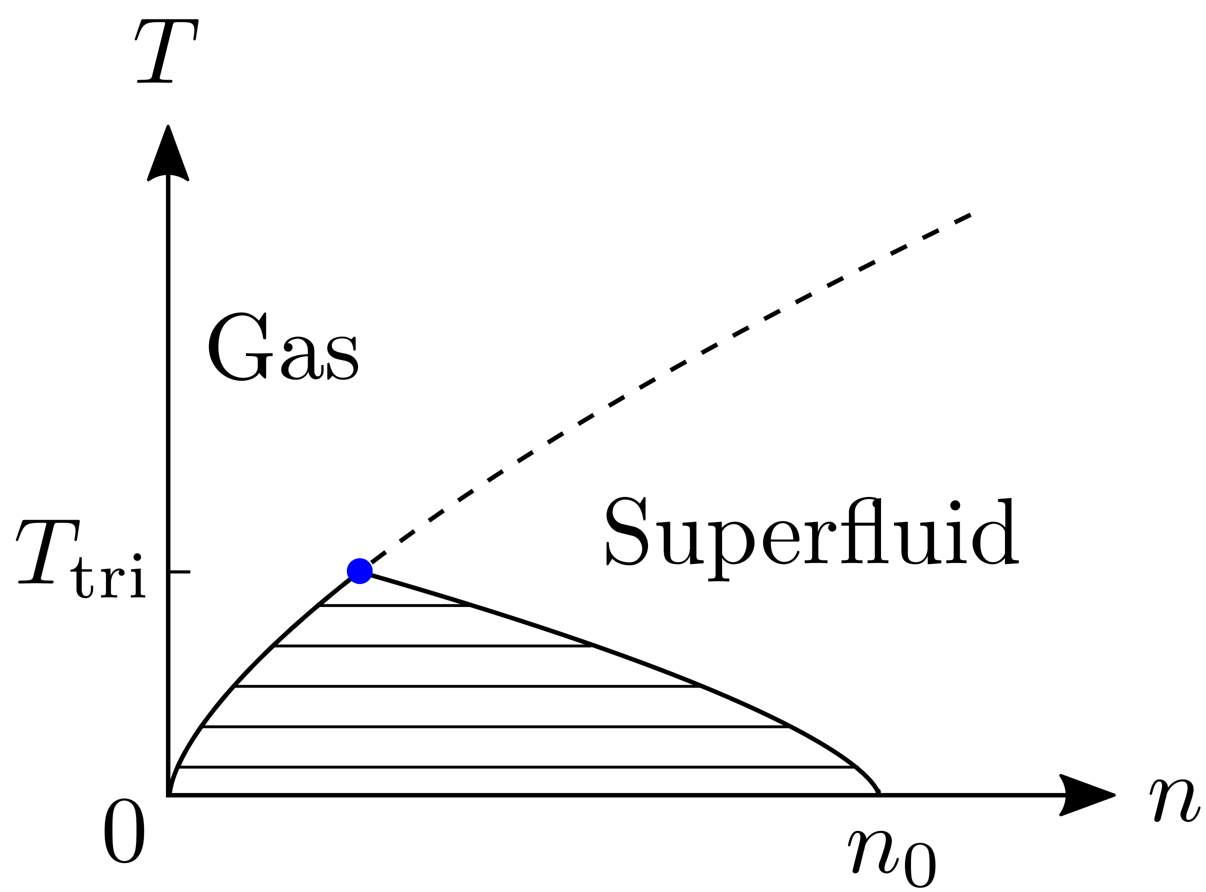


$$\mathcal{L}_{3\mathrm{D}}/\beta = \frac{\hbar^2}{2m_{\mathrm{eff}}}|\boldsymbol{\nabla}\psi_0|^2 - \mu_{\mathrm{eff}}\psi_0^\dagger\psi_0 + \frac{g_{\mathrm{eff}}}{2}(\psi_0^\dagger\psi_0)^2 + \frac{G}{6}(\psi_0^\dagger\psi_0)^3,$$

$$\begin{aligned}
\mu_{\mathrm{eff}} &= \mu - \frac{2g}{\lambda_T^3}L(\beta\mu) - \frac{6\pi g'}{\lambda_T^5}M(\beta\mu) - \frac{3G}{\lambda_T^6}L^2(\beta\mu),\\
g_{\mathrm{eff}} &= g + \frac{3G}{\lambda_T^3}L(\beta\mu),\\
\frac{1}{m_{\mathrm{eff}}} &= \frac{1}{m} + \frac{2g'}{\hbar^2\lambda_T^3}L(\beta\mu)\,.
\end{aligned}$$

$$\begin{aligned}
L(x) &= \mathrm{Li}_{3/2}\left(e^x\right) + \sqrt{-4\pi x}\,,\\
M(x) &= \mathrm{Li}_{5/2}\left(e^x\right) - \frac{4}{3}\sqrt{-\pi x^3}\,,
\end{aligned}$$

Phase diagram



Results of finite-T calculations

- Tricritical point
$$T_{\rm tri} = \frac{2\pi\hbar^2}{m}\left[\frac{4\pi(-a)}{3\zeta(\frac{3}{2})D}\right]^{2/3}$$

$$n_{\rm tri} = \frac{4\pi}{3}\frac{(-a)}{D} = \frac{2}{9}n(T=0)$$

On coexistence curve

$$n_{\rm gas} = \left(\frac{T}{T_{\rm tri}}\right)^{3/2} n_{\rm tri}$$

$$n_{\rm liq} = \left[\frac{9}{2} - \frac{7}{2}\left(\frac{T}{T_{\rm tri}}\right)^{3/2}\right] n_{\rm tri}$$