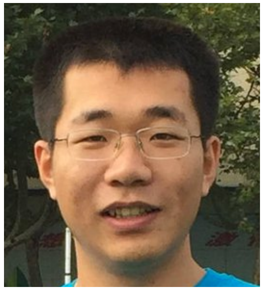

Xiuqi
Ma

Wilbur
Shirley

Meng
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Michael
Levin

John
McGreevy

Fracton and Chern Simons Theory

XIE CHEN, CALTECH

UQM, JUN. 2020

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INSTITUTE FOR QUANTUM INFORMATION AND MATTER

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INSTITUTE

Simons Collaboration on
Ultra-Quantum Matter

Chern-Simons and Fracton

Fractonic Chern-Simons and BF theories, You, Devakul, Sondhi, Burnell, arXiv:1904.11530

Chern-Simons and Fracton

symmetric integer matrix

$$\mathcal{L} = -\frac{1}{4e^2}\sum_I \mathcal{F}^I_{\mu\nu}\mathcal{F}^{I,\mu\nu} + \frac{1}{4\pi}\sum_{IJ} K_{IJ}\epsilon^{\mu\nu\lambda}\mathcal{A}^I_\mu\partial_\nu\mathcal{A}^J_\lambda$$

2+1D

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$K = 1$ Integer quantum Hall

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2+1D

$K = 1$ Integer quantum Hall

$K = 3$ Fractional quantum Hall

$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ Z_2 gauge theory

Chern-Simons and Fracton

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3+1D $\quad I, J = ... - 1, 0, 1, 2, ...$

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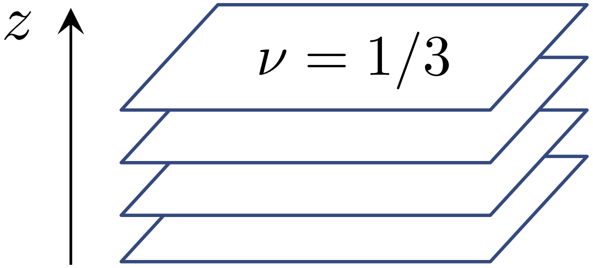

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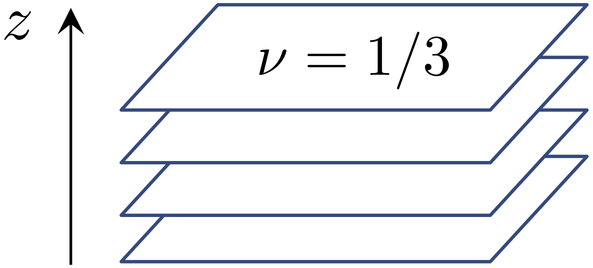
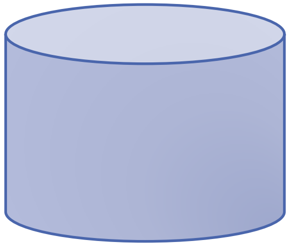
- Ground state degeneracy 3^N exponential in height in z
- Quasi-particles move in xy planes only – planons
- Entanglement has a sub-leading term linear in height in z





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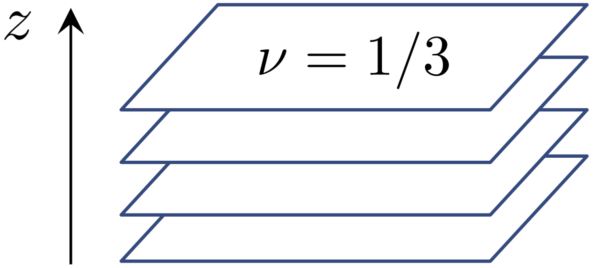

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Trivial type of Fracton
Anything more nontrivial?

Tri-diagonal K matrix

$$K(131) = \begin{pmatrix} 3 & 1 & & & 1 \\ 1 & 3 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 3 & 1 \\ 1 & & & 1 & 3 \end{pmatrix}_{N\times N} .$$

Qiu, Joynt, MacDonald, 1990; Naud, Pryadko, Sondhi, 2000

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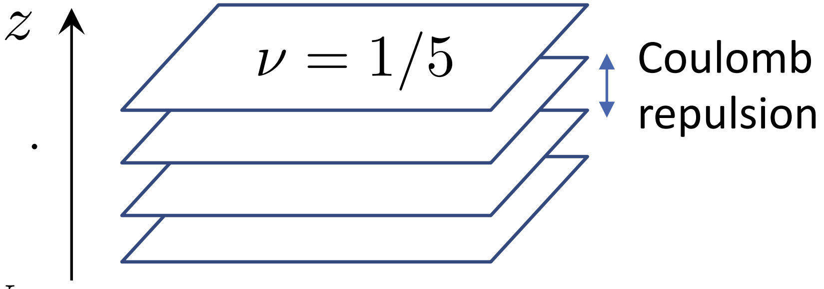

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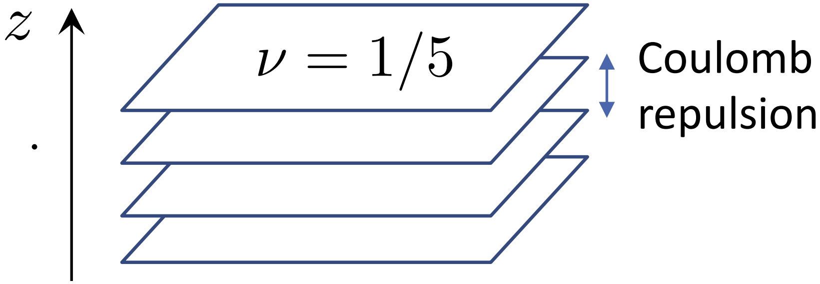

$N \to \infty$ Irrational statistics

$$\theta_{IJ} = \frac{1}{\sqrt{5}} \left(\frac{\sqrt{5}-3}{2} \right)^{|I-J|}$$

Qiu, Joynt, MacDonald, 1990; Naud, Pryadko, Sondhi, 2000

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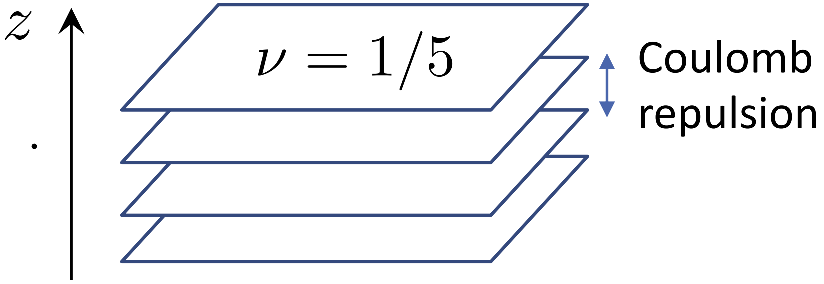

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$$(3 + \sqrt{5})^N + (3 - \sqrt{5})^N - 2(-1)^N$$
$$\sim (3 + \sqrt{5})^N$$

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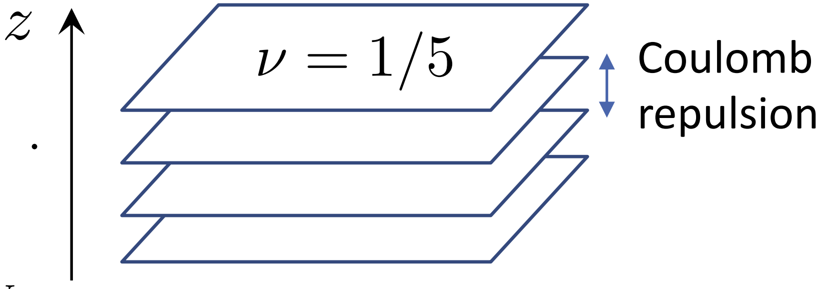

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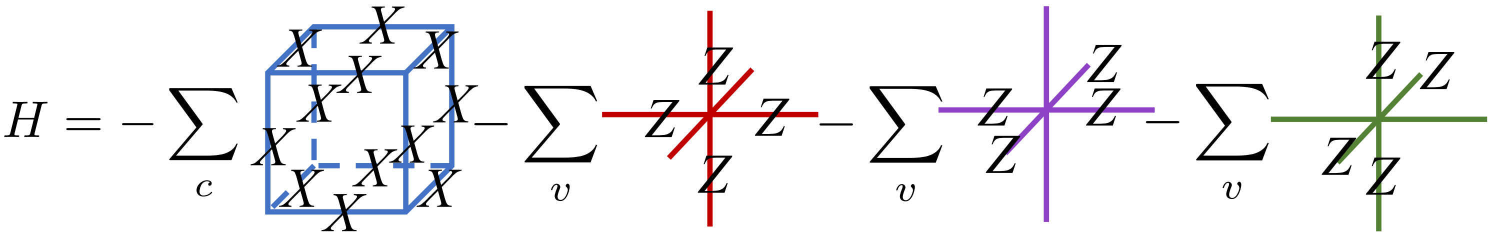
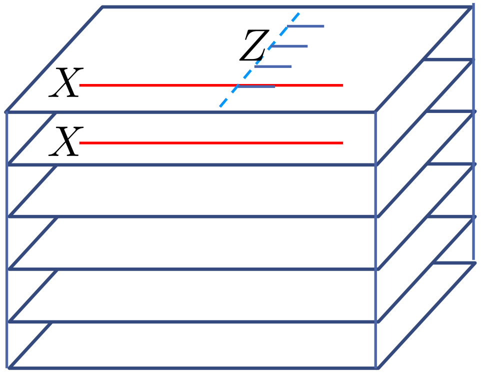
Planon Fusion group $G_N = \mathbb{Z}_{F_N} \times \mathbb{Z}_{5F_N}$

F_N Nth number in the Fibonacci sequence

Qiu, Joynt, MacDonald, 1990; Naud, Pryadko, Sondhi, 2000

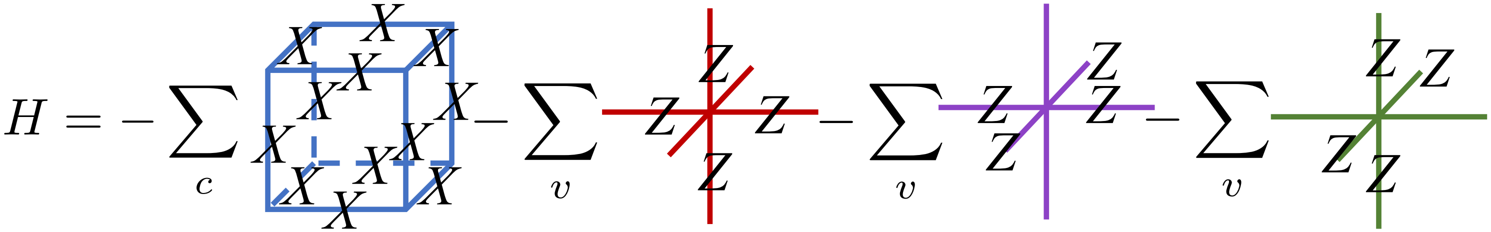
Compare to X-cube





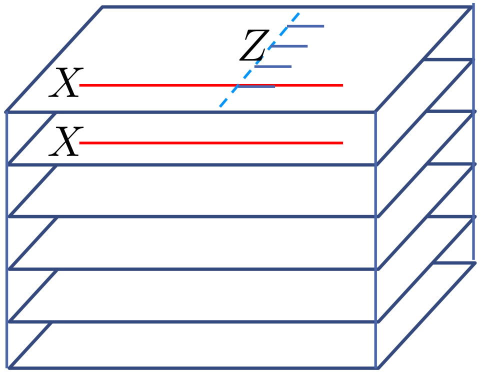
Vijay, Haah, Fu, 16

Compare to X-cube



Planon Fusion group

$$G_N = (\mathbb{Z}_2^X)^{N-1} \times (\mathbb{Z}_2^Z)^{N-1}$$



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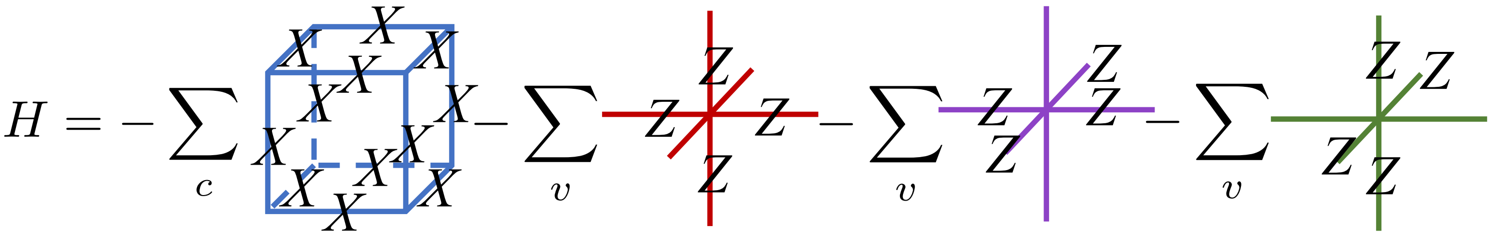

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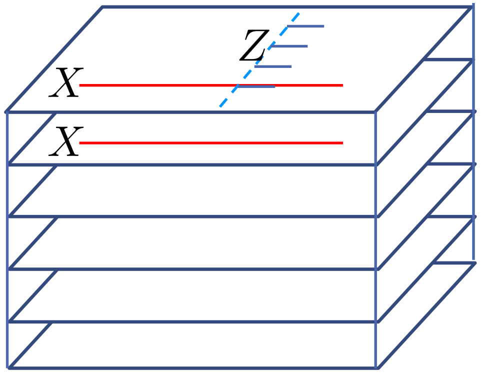
Statistics $\theta_{X_{i-1/2},Z_i} = \theta_{Z_i,X_{i+1/2}} = \pi$



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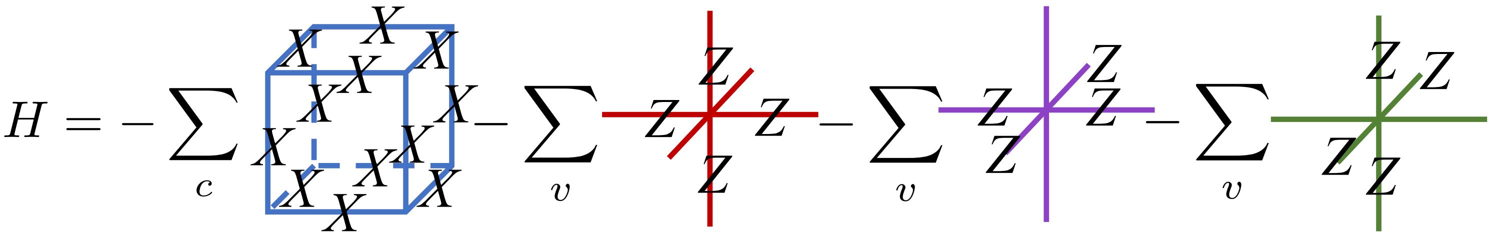

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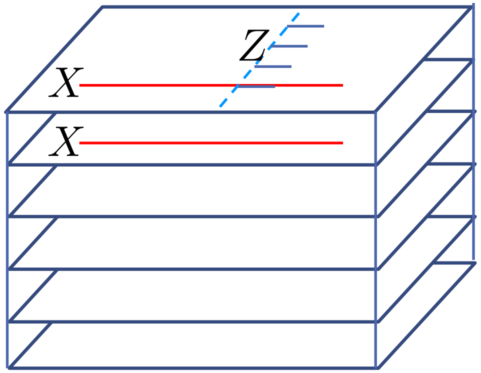
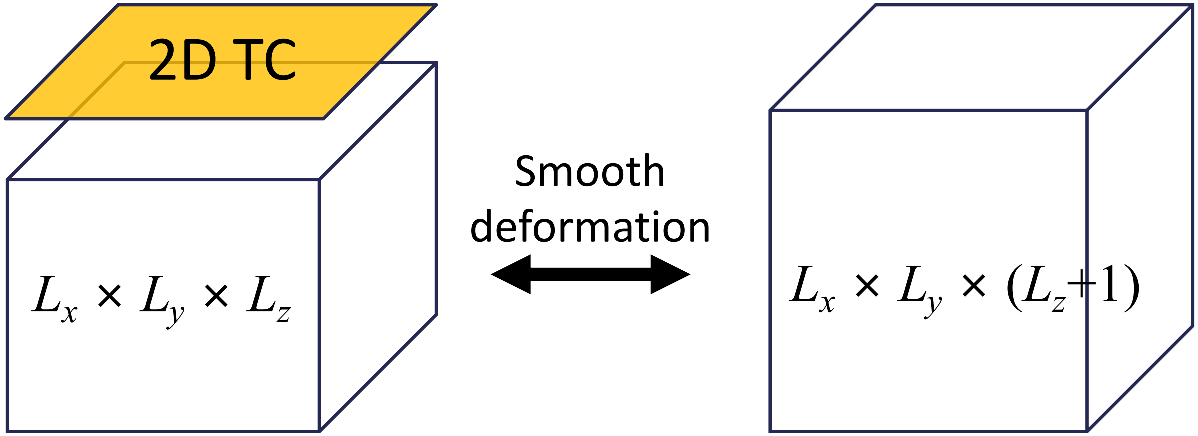
Ground state degeneracy $\sim 2^{2N}$



Vijay, Haah, Fu, 16

Compare to X-cube

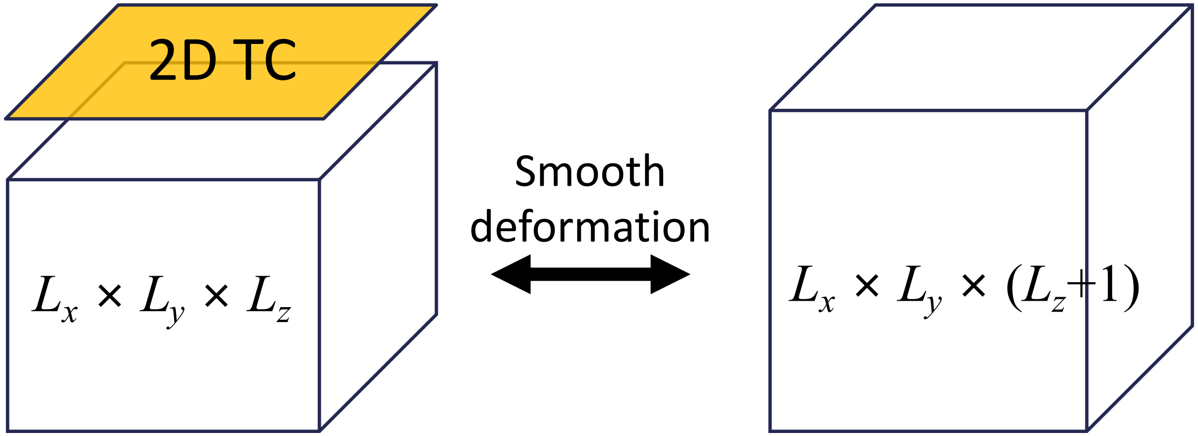
Foliation



Shirley, Slagle, Wang, Chen, 18

Compare to X-cube

Foliation



- Exact exponential scaling of ground state degeneracy
- Finite order of planons
- Short range (no tail) statistics
- K(131) is not foliated

Shirley, Slagle, Wang, Chen, 18

Foliated K matrix

$$K = \begin{array}{ccccccccc} & e_1 & m_1 & e_2 & m_2 & e_3 & m_3 & e_4 & \\ \ddots & & & & & & & & \\ & 0 & 2 & -1 & & & & & \\ & 2 & 0 & & & & & & \\ & -1 & & 0 & 2 & -1 & & & \\ & & & 2 & 0 & & & & \\ & & & -1 & & 0 & 2 & -1 & \\ & & & & & 2 & 0 & & \\ & & & & & -1 & & 0 & \\ & & & & & & & & \ddots \end{array} .$$

$$K^{-1} = \frac{1}{4} \begin{array}{ccccccccc} & m_0 & e_1 & m_1 & e_2 & m_2 & e_3 & m_3 & \\ \ddots & & & & & & & & \\ & 0 & & 1 & & & & & \\ & & 0 & 2 & & & & & \\ & 1 & 2 & 0 & & 1 & & & \\ & & & & 0 & 2 & & & \\ & & & 1 & 2 & 0 & & 1 & \\ & & & & & & 0 & 2 & \\ & & & & & 1 & 2 & 0 & \\ & & & & & & & & \ddots \end{array} .$$

Shirley, Slagle, Chen, arXiv:1907.09048

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$$K(131) = \begin{pmatrix} 3 & 1 & & & & & 1 \\ 1 & 3 & 1 & & & & \\ & 1 & 3 & 1 & & & \\ & & 1 & 3 & 1 & & \\ & & & 1 & 3 & 1 & \\ & & & & 1 & 3 & 1 \\ 1 & & & & & 1 & 3 \end{pmatrix}$$

$$K(131)^{-1} = \frac{1}{65} \begin{pmatrix} 29 & -11 & 4 & -1 & -1 & 4 & -11 \\ -11 & 29 & -11 & 4 & -1 & -1 & 4 \\ 4 & -11 & 29 & -11 & 4 & -1 & -1 \\ -1 & 4 & -11 & 29 & -11 & 4 & -1 \\ -1 & -1 & 4 & -11 & 29 & -11 & 4 \\ 4 & -1 & -1 & 4 & -11 & 29 & -11 \\ -11 & 4 & -1 & -1 & 54 & -11 & 29 \end{pmatrix}$$

Shirley, Slagle, Chen, arXiv:1907.09048

Foliated K matrix

$$W = \begin{array}{c} \\ \\ e_1 \\ m_1 \\ e_2 \\ m_2 \\ e_3 \\ m_3 \\ e_4 \\ m_4 \\ e_5 \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccccc} \tilde{e}_1 & \tilde{m}_1 & \tilde{e}^A & \tilde{m}^A & \tilde{e}^B & \tilde{m}^B & \tilde{e}_2 & \tilde{m}_2 & \tilde{e}_3 \end{array} \\ \left(\begin{array}{ccccccccccc} \ddots & & & & & & & & & & \\ & 1 & & & & & & & & & \\ & & 1 & & & & 1 & & & & \\ & & & 1 & & & & -1 & & & \\ & & & & 1 & & & & & & \\ & -1 & & & & 1 & & & & & \\ & & & & & & 1 & & & & \\ & & & & & & & 1 & & & \\ & -1 & & & 1 & & & & 1 & & \\ & & & & & & & & & 1 & \\ & & & & & & & & & & \ddots \end{array}\right) \end{array}. \quad W^T K W = \begin{array}{c} \begin{array}{ccccccccc} \tilde{e}_1 & \tilde{m}_1 & \tilde{e}^A & \tilde{m}^A & \tilde{e}^B & \tilde{m}^B & \tilde{e}_2 & \tilde{m}_2 & \tilde{e}_3 \end{array} \\ \left(\begin{array}{ccccccccccc} \ddots & & & & & & & & & & \\ & 0 & 2 & & & & & -1 & & & \\ & 2 & 0 & & & & & & & & \\ & & & 0 & 2 & -1 & 0 & & & & \\ & & & 2 & 0 & 0 & 0 & & & & \\ & & & -1 & 0 & 0 & 2 & & & & \\ & & & 0 & 0 & 2 & 0 & & & & \\ & -1 & & & & & & 0 & 2 & -1 & \\ & & & & & & & 2 & 0 & & \\ & & & & & & & -1 & & 0 & \\ & & & & & & & & & & \ddots \end{array}\right) \end{array}.$$

Shirley, Slagle, Chen, arXiv:1907.09048

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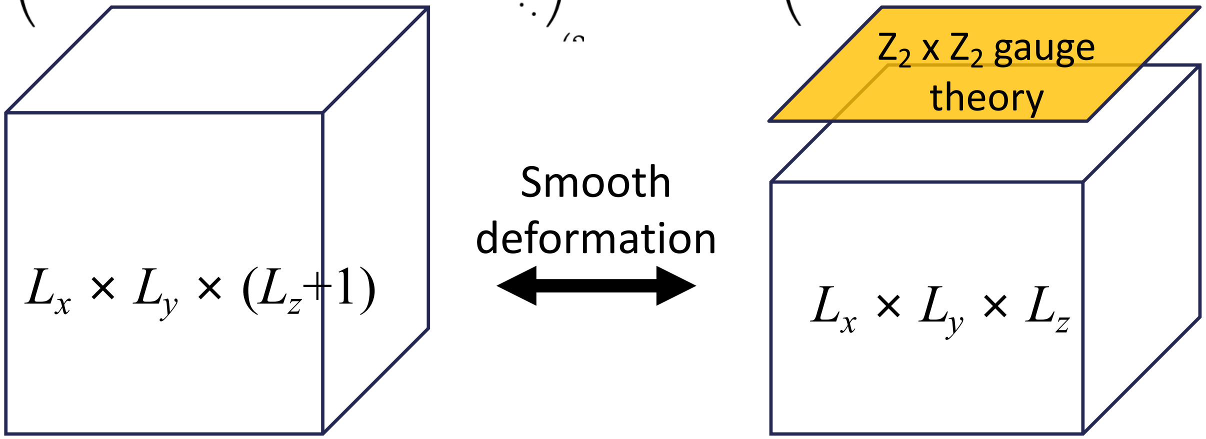
$$W = \begin{array}{c|ccccccccc} & \tilde{e}_1 & \tilde{m}_1 & \tilde{e}^A & \tilde{m}^A & \tilde{e}^B & \tilde{m}^B & \tilde{e}_2 & \tilde{m}_2 & \tilde{e}_3 \\ \hline & \ddots & & & & & & & & \\ e_1 & 1 & & & & & & & & \\ m_1 & & 1 & & & & 1 & & & \\ e_2 & & & 1 & & & & -1 & & \\ m_2 & & & & 1 & & & & & \\ e_3 & -1 & & & & 1 & & & & \\ m_3 & & & & & & 1 & & & \\ e_4 & & & & & & & 1 & & \\ m_4 & -1 & & & 1 & & & & 1 & \\ e_5 & & & & & & & & & 1 \\ & & & & & & & & & \ddots \end{array}. \quad W^T K W = \begin{array}{ccccccccc} \tilde{e}_1 & \tilde{m}_1 & \tilde{e}^A & \tilde{m}^A & \tilde{e}^B & \tilde{m}^B & \tilde{e}_2 & \tilde{m}_2 & \tilde{e}_3 \\ \hline \ddots & & & & & & & & \\ 0 & 2 & & & & & -1 & & \\ 2 & 0 & & & & & & & \\ & & 0 & 2 & -1 & 0 & & & \\ & & 2 & 0 & 0 & 0 & & & \\ & & -1 & 0 & 0 & 2 & & & \\ & & 0 & 0 & 2 & 0 & & & \\ -1 & & & & & & 0 & 2 & -1 \\ & & & & & & 2 & 0 & \\ & & & & & & -1 & & 0 \\ & & & & & & & & \ddots \end{array}.$$



Shirley, Slagle, Chen, arXiv:1907.09048

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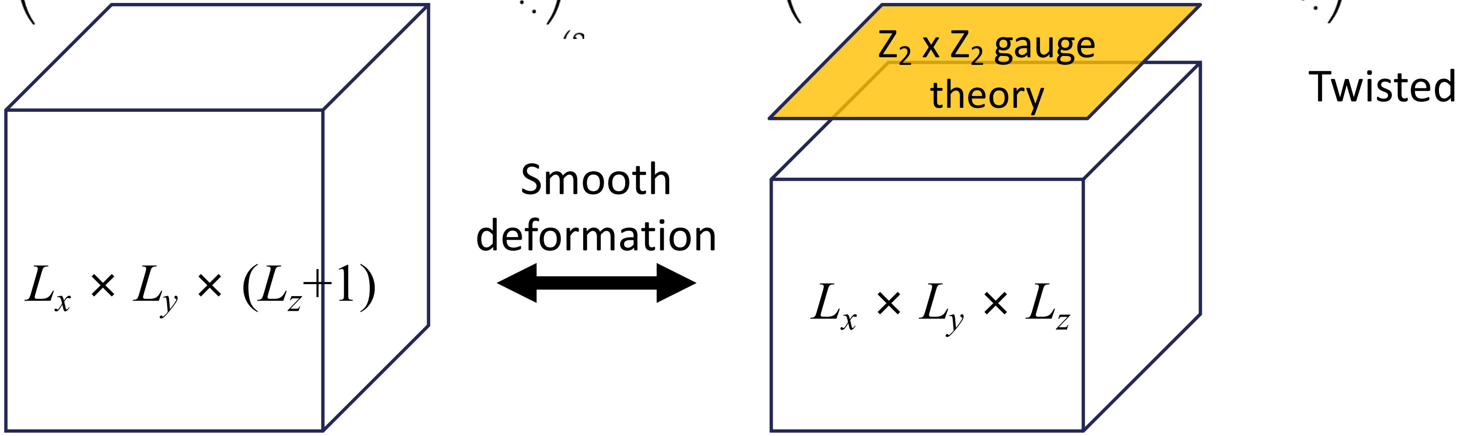
$$W = \begin{array}{c|ccccccccccc} & & \tilde e_1 & \tilde m_1 & \tilde e^A & \tilde m^A & \tilde e^B & \tilde m^B & \tilde e_2 & \tilde m_2 & \tilde e_3 & \\ \hline & \ddots & & & & & & & & & & \\ e_1 & & 1 & & & & & & & & & \\ m_1 & & & 1 & & & & 1 & & & & \\ e_2 & & & & 1 & & & & -1 & & & \\ m_2 & & & & & 1 & & & & & & \\ e_3 & & -1 & & & & 1 & & & & & \\ m_3 & & & & & & & 1 & & & & \\ e_4 & & & & & & & & 1 & & & \\ m_4 & & -1 & & & 1 & & & & 1 & & \\ e_5 & & & & & & & & & & 1 & \\ & & & & & & & & & & & \ddots \end{array} \,.\quad W^T K W = \begin{array}{ccccccccccc} & \tilde e_1 & \tilde m_1 & \tilde e^A & \tilde m^A & \tilde e^B & \tilde m^B & \tilde e_2 & \tilde m_2 & \tilde e_3 & \\ \hline \ddots & & & & & & & & & & \\ & 0 & 2 & & & & & -1 & & & \\ & 2 & 0 & & & & & & & & \\ & & & 0 & 2 & -1 & 0 & & & & \\ & & & 2 & 0 & 0 & 0 & & & & \\ & & & -1 & 0 & 0 & 2 & & & & \\ & & & 0 & 0 & 2 & 0 & & & & \\ & -1 & & & & & & 0 & 2 & -1 & \\ & & & & & & & 2 & 0 & & \\ & & & & & & & -1 & & 0 & \\ & & & & & & & & & & \ddots \end{array} \,.$$



Z_2 x Z_2 gauge theory

Twisted

Smooth deformation

$L_x \times L_y \times (L_z+1)$

$L_x \times L_y \times L_z$

Shirley, Slagle, Chen, arXiv:1907.09048

∞-d K matrix and Fracton

∞-d K matrix and Fracton

- Beyond exactly solvable models

∞-d K matrix and Fracton

- Beyond exactly solvable models
- For foliated models
  - Identify examples
  - Calculate properties
  - Study equivalence up to foliation using K matrix formulation

∞-d K matrix and Fracton

- Beyond exactly solvable models
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  - Study equivalence up to foliation using K matrix formulation
- For non-foliated models
  - Identify examples
  - Calculate properties
  - How to interpret their structure?

But first

Question:

- Do they represent local 3D model?
- Are they gapped or gapless?
- form of string operator

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Answer:

- In terms of Laughlin wave function (Qiu, Joynt, MacDonald, 1990; Naud, Pryadko, Sondhi, 2000)

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- Do they represent local 3D model?
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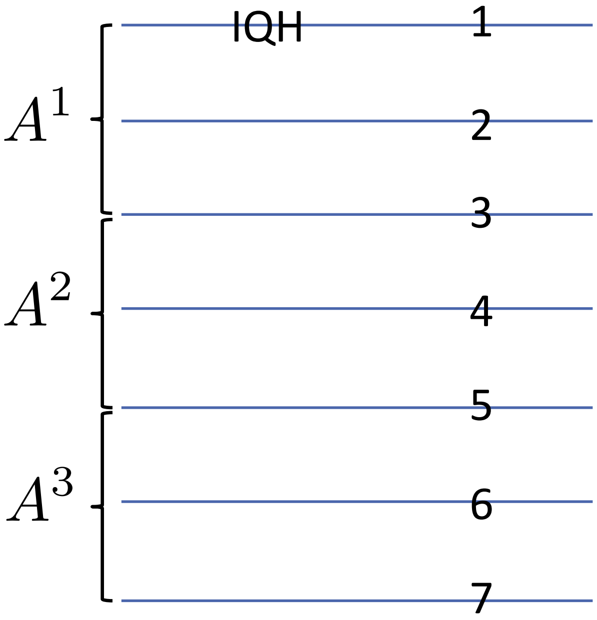
Answer:

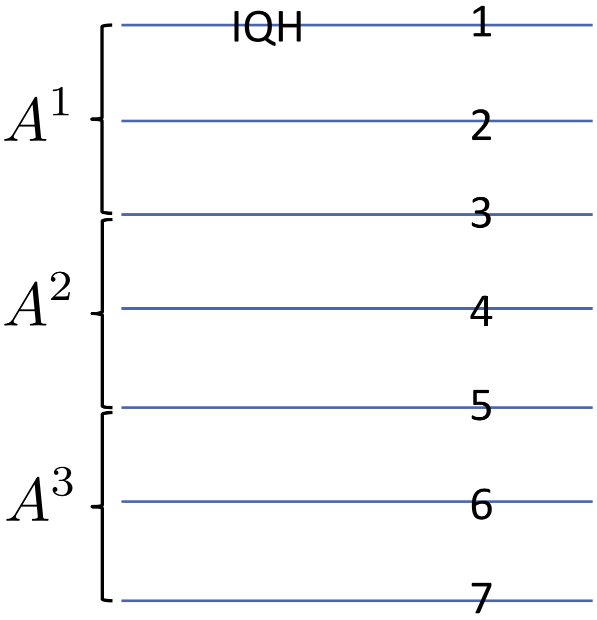
- In terms of Laughlin wave function (Qiu, Joynt, MacDonald, 1990; Naud, Pryadko, Sondhi, 2000)
- For quasi-diagonal K matrix
  - Lattice realization
  - String operator
  - Spectrum (field theory calculation)

Lattice realization



- Stack of IQH
- Coupled to planar U(1) symmetries
- Gauge the U(1) symmetries

Lattice realization



- Stack of IQH
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- Gauge the U(1) symmetries

$$\mathcal{L} = \frac{1}{4\pi}\sum_{l}\epsilon^{\mu\nu\lambda}a^l_\mu\partial_\nu a^l_\lambda +$$

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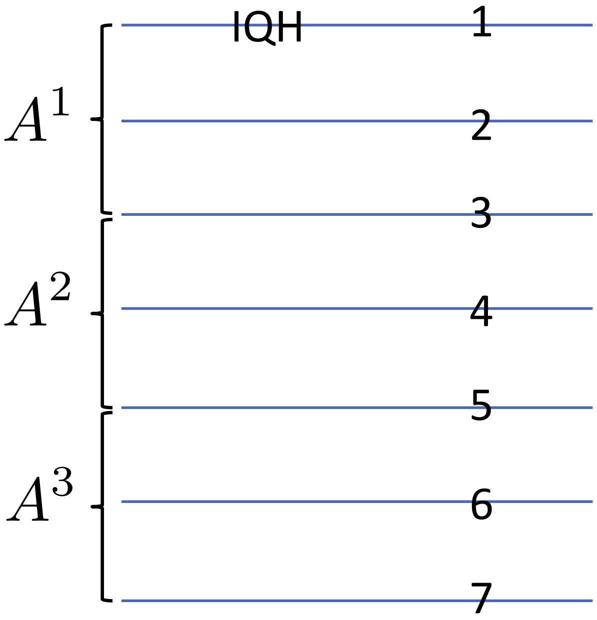
Lattice realization



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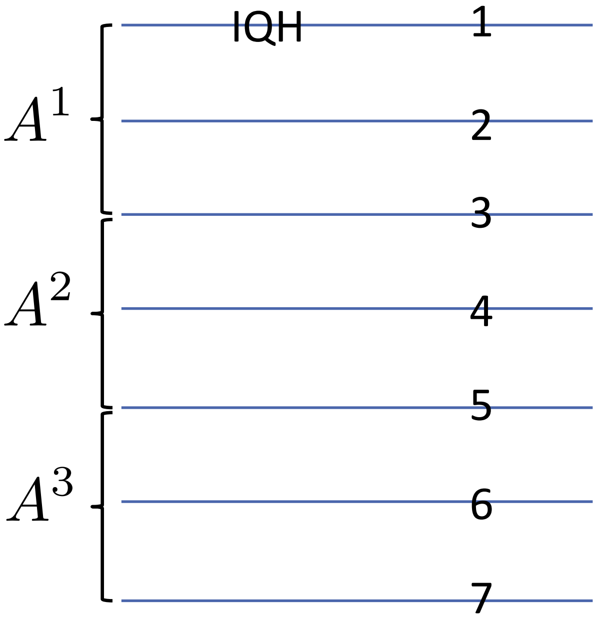

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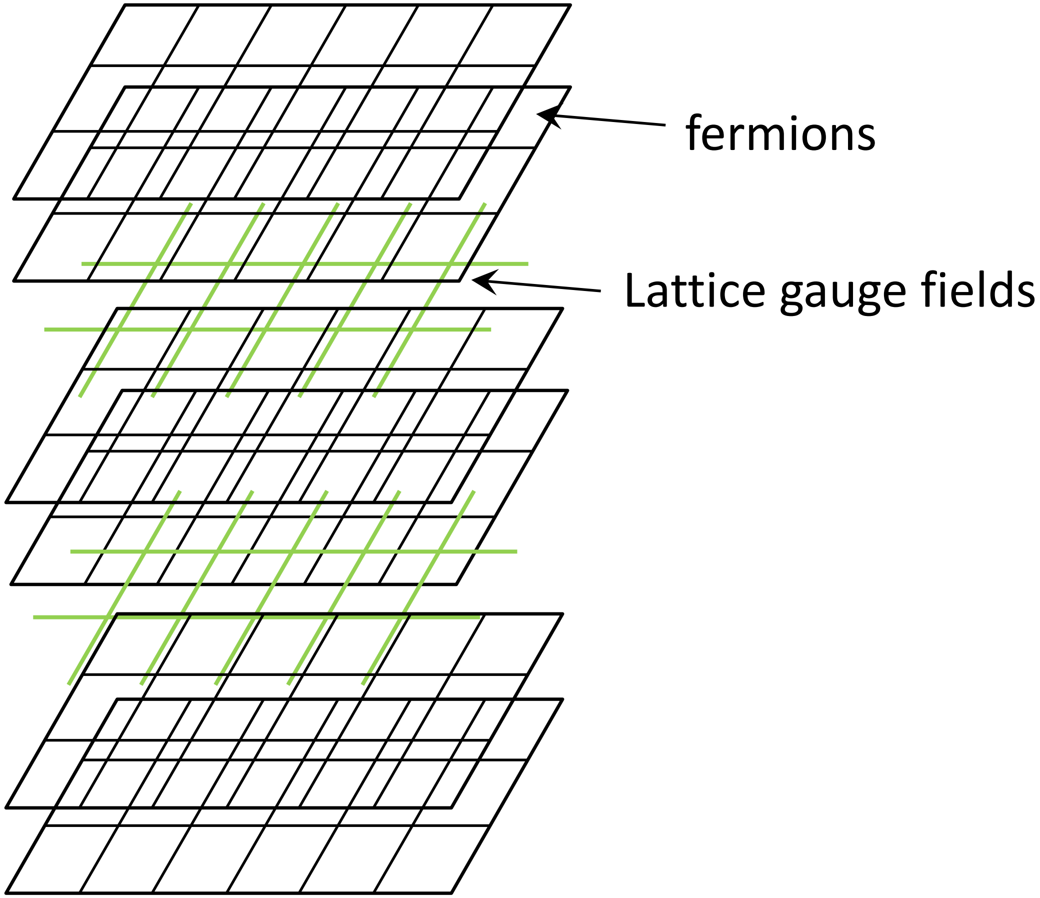
$$H = \sum_l \sum_{\langle \mathbf{r}\mathbf{r}'\rangle} u_{l,\mathbf{r}\mathbf{r}'} \exp\left(i\sum_i C^{il} A^i_{\mathbf{r}\mathbf{r}'}\right) c^\dagger_{l,\mathbf{r}'} c_{l,\mathbf{r}} + \sum_i \left[\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} g_E \left(E^i_{\mathbf{r}\mathbf{r}'}\right)^2 - \sum_p \cos B^i_p + \lambda \sum_{\mathbf{r}} \left(Q^i_{\mathbf{r}}\right)^2\right]$$

Fermion hopping

Gauge symmetry

$$Q^i_{\mathbf{r}} = (\nabla \cdot \mathbf{E})^i_{\mathbf{r}} - \sum_l C^{il} c^\dagger_{l,\mathbf{r}} c_{l,\mathbf{r}}$$

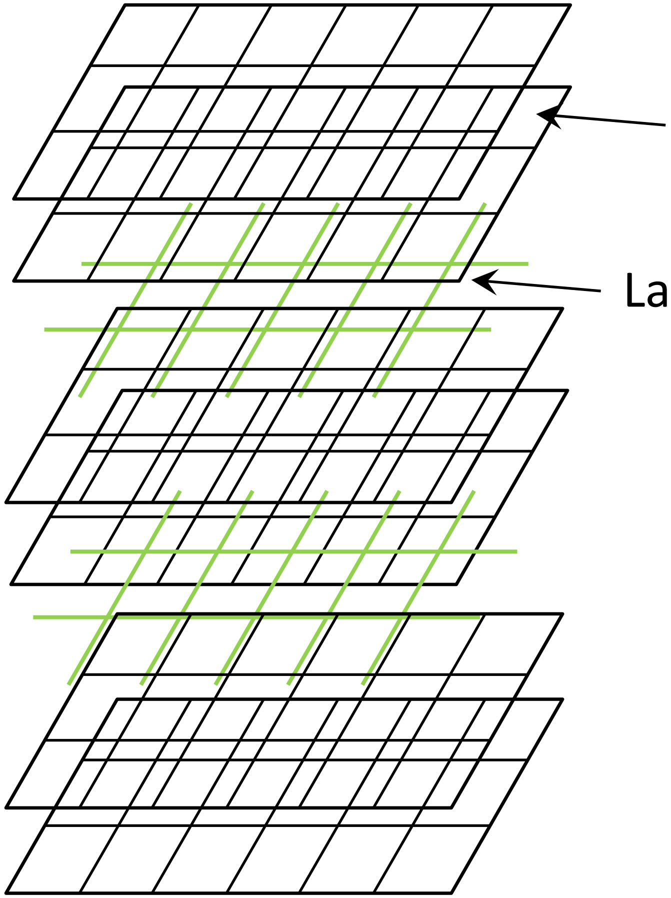
String operator



fermions

Lattice gauge fields

String operator



fermions

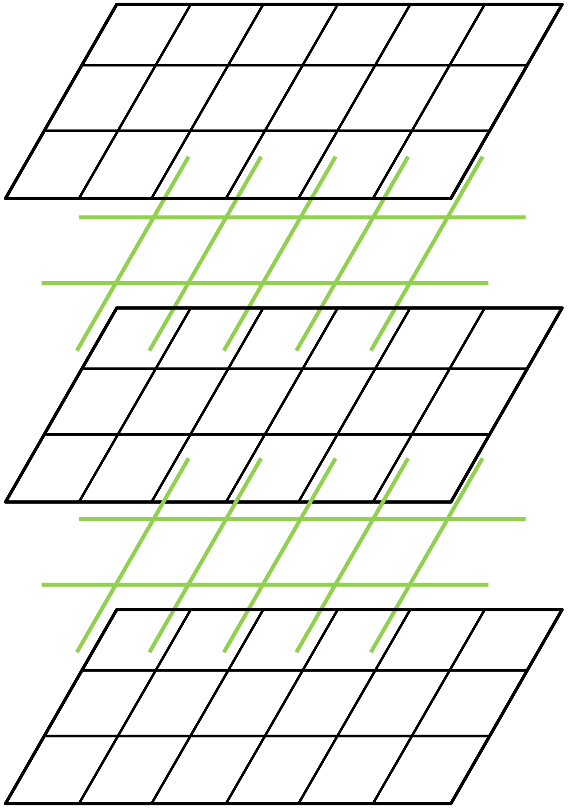
Lattice gauge fields

In terms of CS gauge field

$$\mathcal{W}_i = e^{i \int dl \mathcal{A}_i}$$

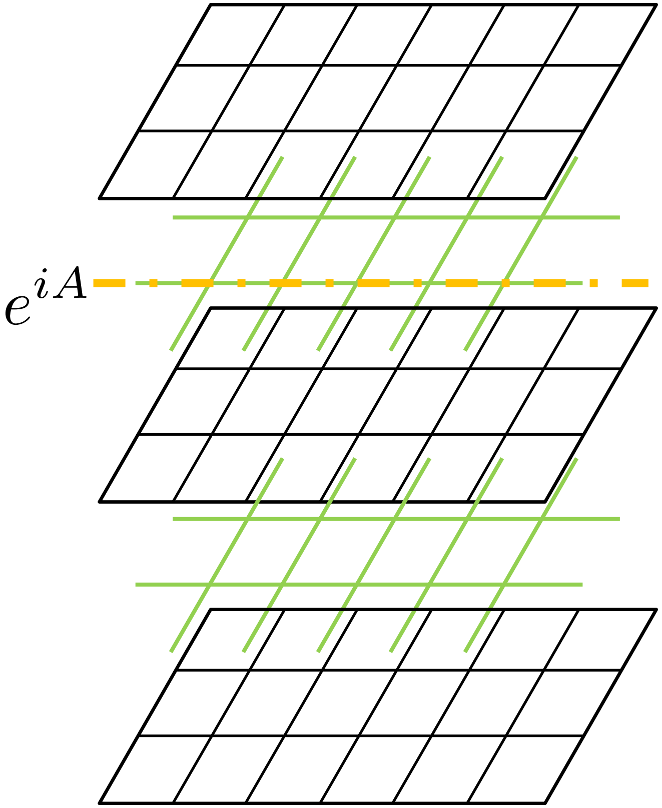
In terms of lattice DOF?

String operator



Charge vector (1,0,0,…)

String operator



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In terms of lattice DOF

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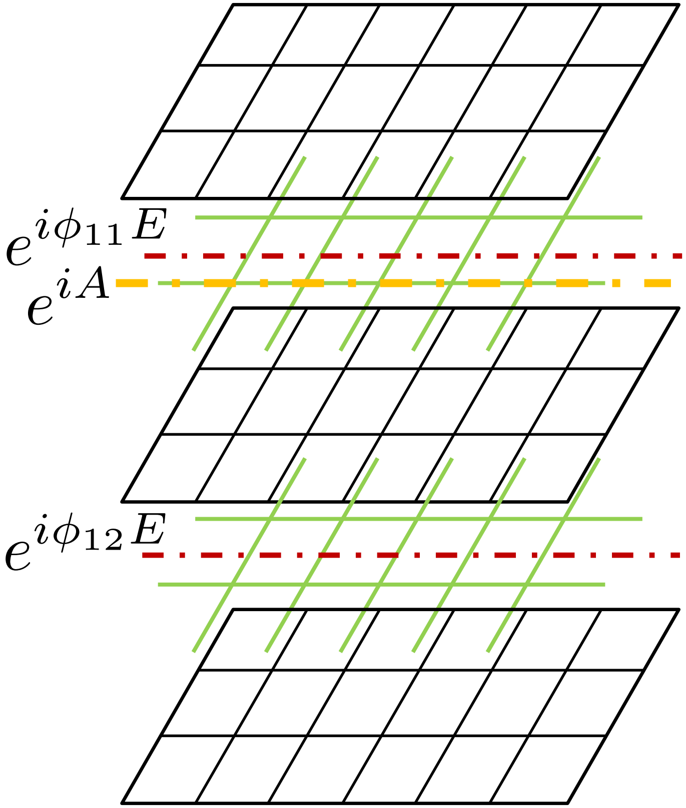

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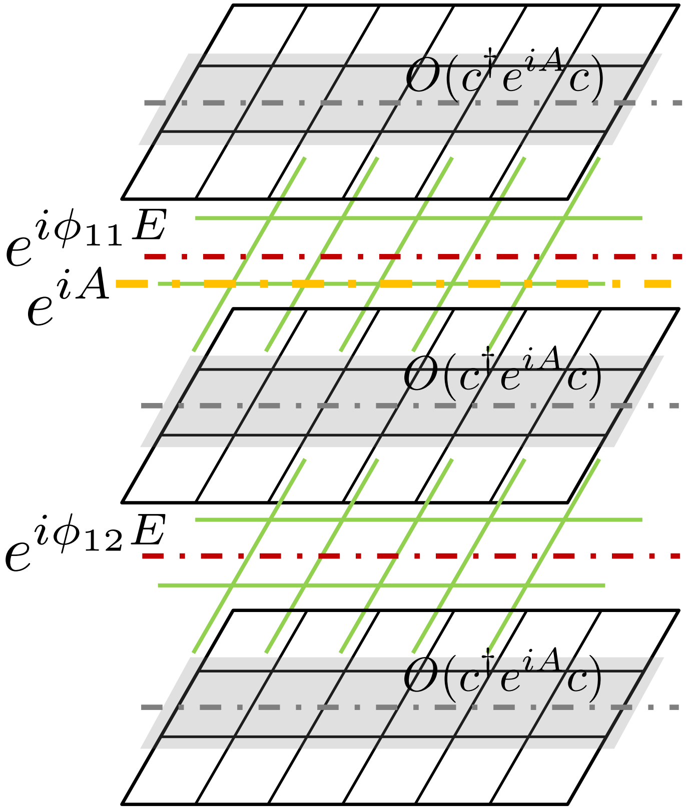
String operator



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- $O(c^\dagger e^{iA} c)$ adiabatically evolves the matter field

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Gapless fracton?

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- What about non-quasi-diagonal ones (e.g. for X-cube)?