Quantum Hall network models as Floquet topological insulators

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Summary

- **Chalker-Coddington network model** used since 1980s to describe transition in the integer quantum Hall effect (IQHE) can be reinterpreted in the modern language as a **Floquet topological insulator.**

- Within the Floquet framework, **it lacks Chern number** instead being characterized by the **Chiral Floquet topological invariant** and thus seemingly belongs to a different class of topological insulators, Chiral Floquet insulators instead of Chern insulators where IQHE belongs.

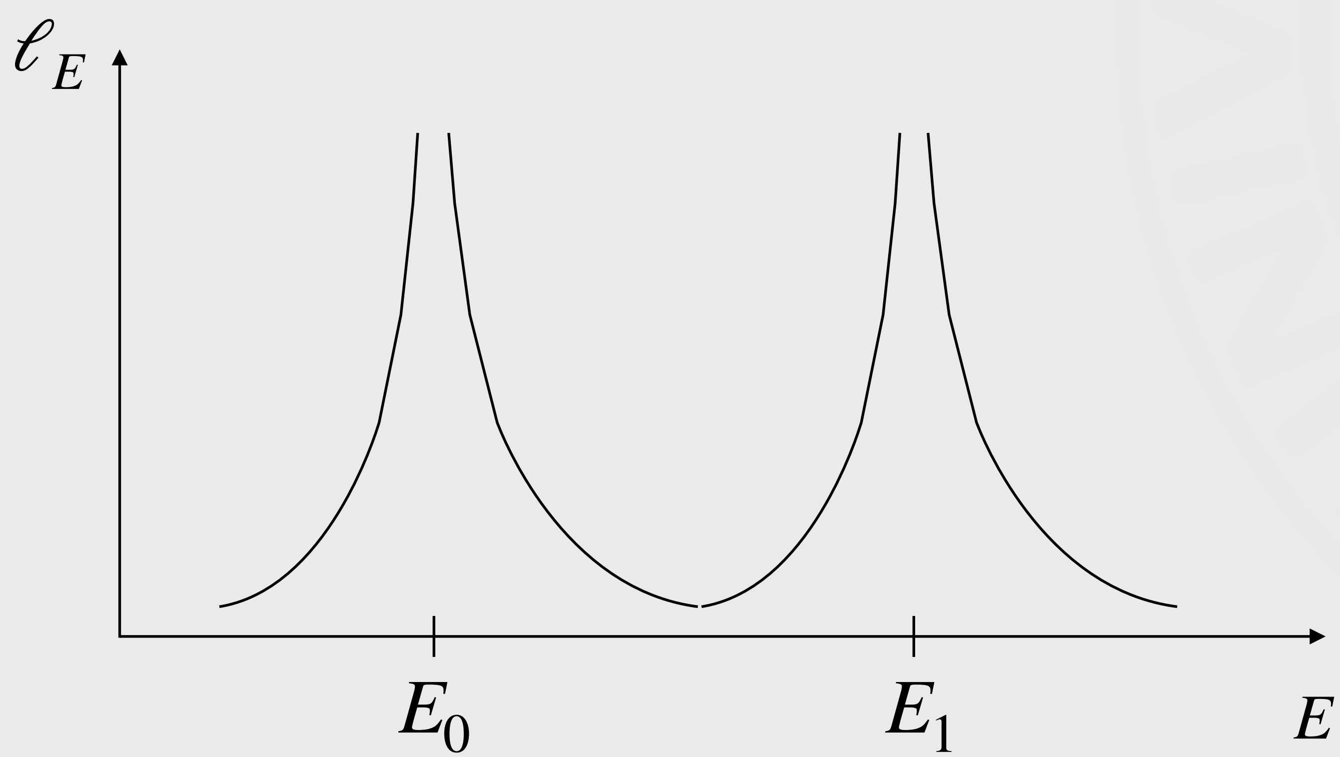
- We show that network model can be used to establish **a map from chiral Floquet insulators to Chern insulators,** so they are more closely related than previously thought.

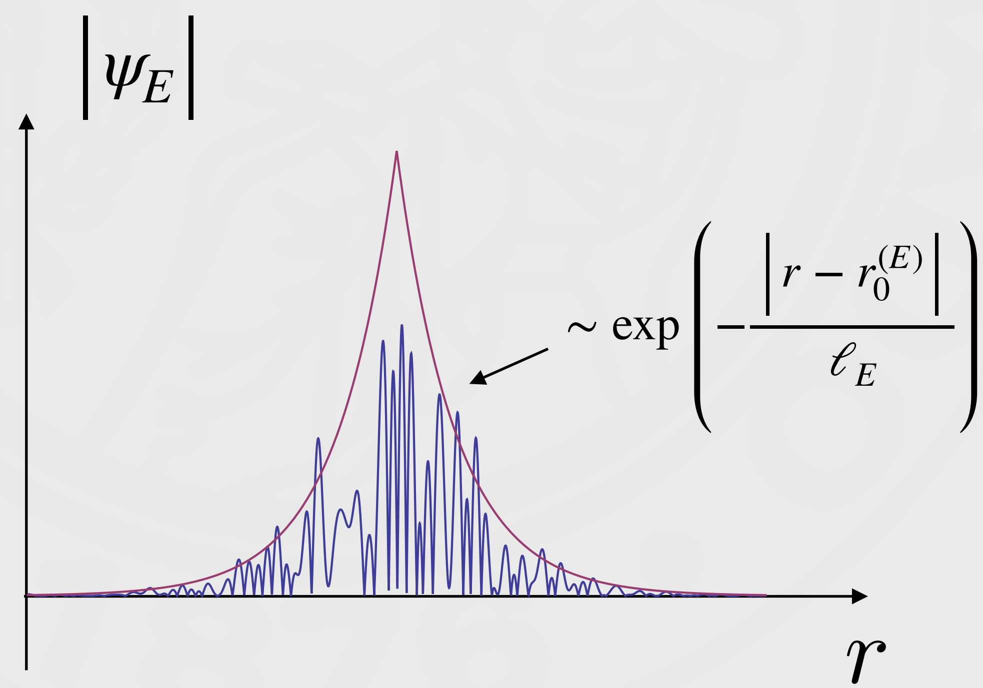
Plateau transition in IQHE

Particle in a magnetic field in a random potential

$$H = \frac{1}{2m}\left[\left(-i\frac{\partial}{\partial x} + \frac{eB}{c}y\right)^2 - \frac{\partial^2}{\partial y^2}\right]\psi + V\psi = E\psi$$

Transition properties encoded in the eigenstates of this Schrödinger equation





$$\ell_E \sim |E - E_0|^{-\nu}$$

$\nu \approx 2.35$

Long-held belief

$\nu \approx 2.6$

K. Slevin and T. Ohtsuki, (2009)

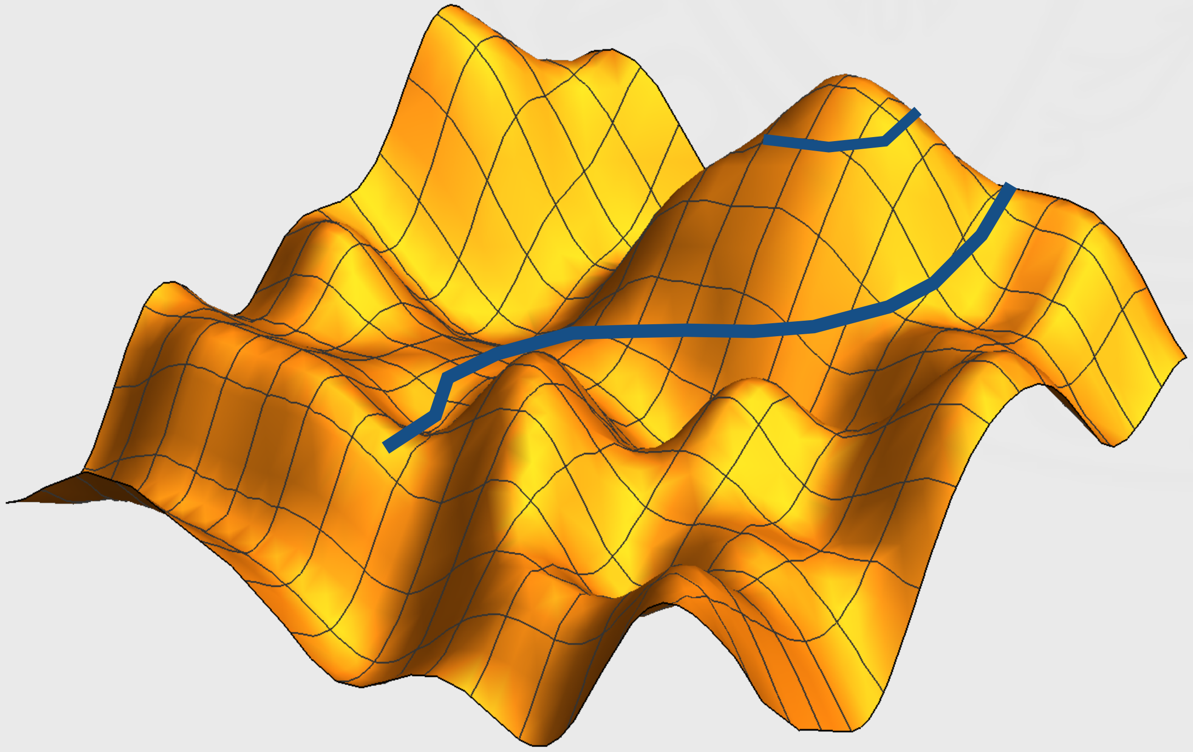
Quasiclassical picture

$$H = \frac{1}{2m}\left[\left(-i\frac{\partial}{\partial x} + \frac{eB}{c}y\right)^2 - \frac{\partial^2}{\partial y^2}\right]\psi + V\psi = E\psi \longrightarrow m a_i = -\frac{\partial V}{\partial x_i} + \frac{eB}{c}\epsilon_{ij}v_j$$

Lowest Landau level $m \to 0$

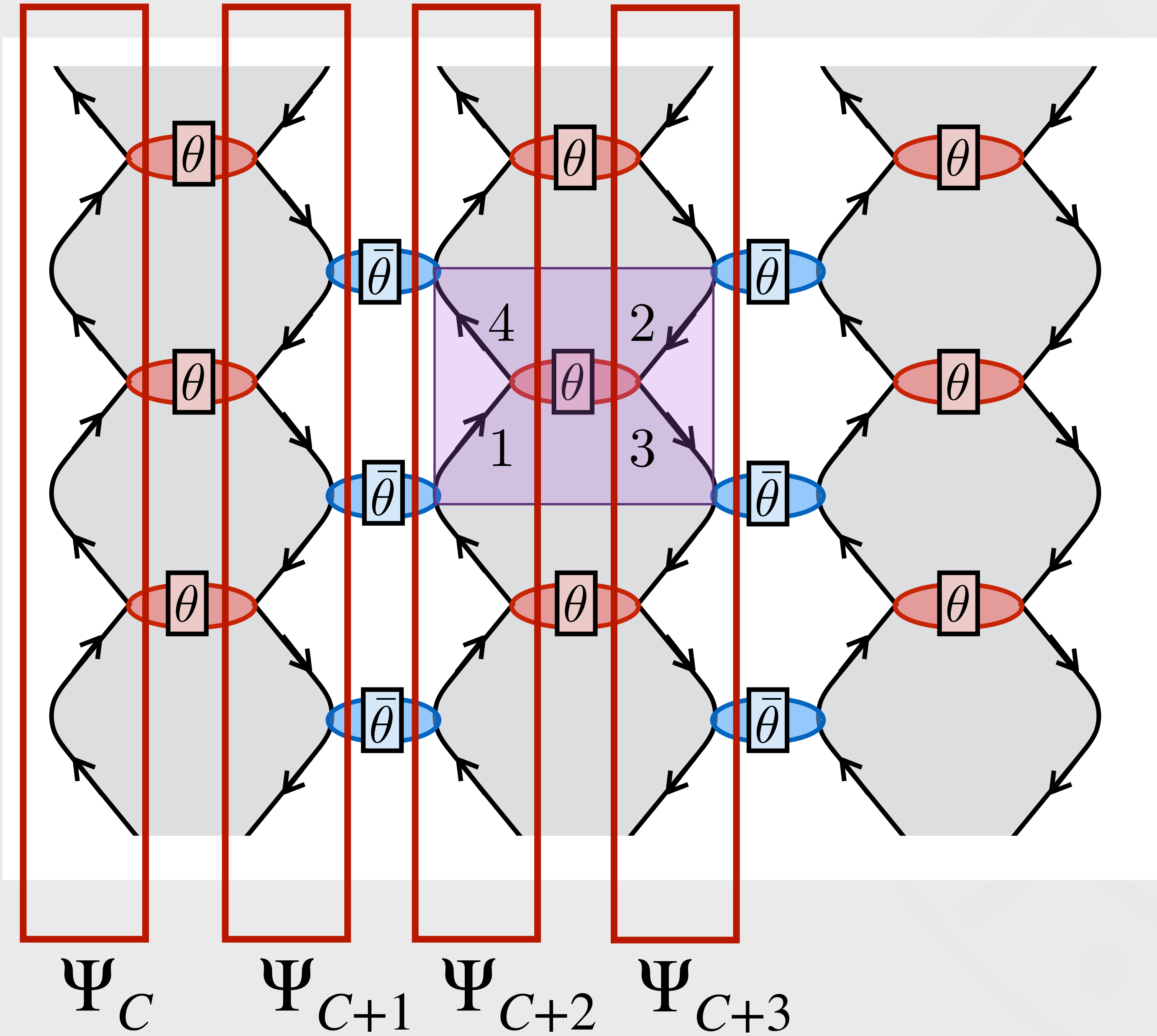
$$\frac{\partial V}{\partial x_i} = \frac{eB}{c}\epsilon_{ij}v_j$$

Movement along equipotential lines



Similar to percolation, but tunneling between nearby trajectories is allowed.

Chalker-Coddington network model



$$\Psi_{C+2} = \underbrace{T_{C+1}(\bar{\theta})T_C(\theta)}_{\text{transfer matrix}}\Psi_C$$

transfer matrix

localization length extracted from the smallest eigenvalue of the transfer matrix.

$$\bar{\theta} = \pi - \theta$$

$$\begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} e^{i\varphi_3} & 0 \\ 0 & e^{i\varphi_4} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

random energy-dependent phases

node scattering

Network model as a chiral Floquet

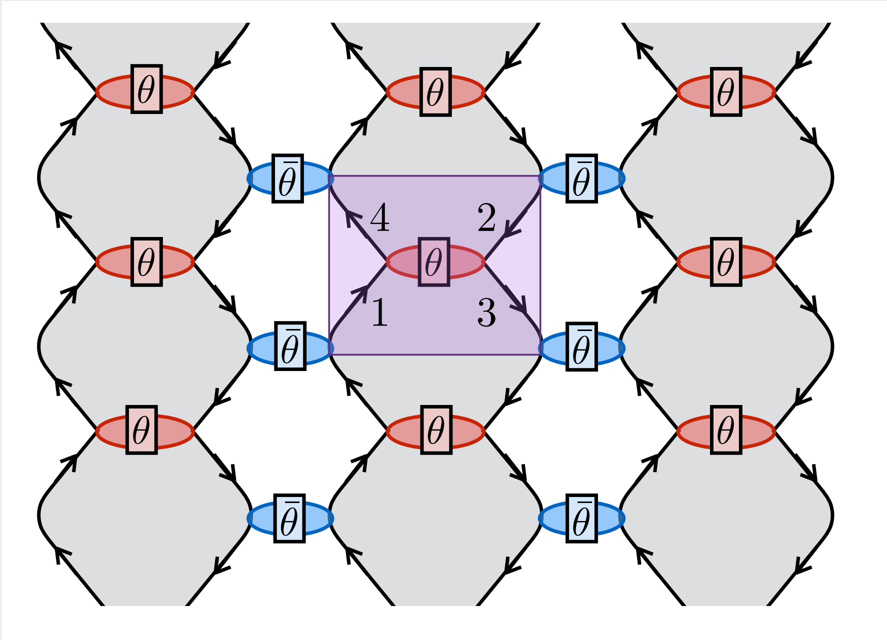
Stroboscopic Floquet evolution

$$\psi(t+1) = U\psi(t)$$

Eigenvalues of U:

$$\lambda = e^{-i\epsilon}$$

$$U(\theta, \vec{k}) = \begin{pmatrix} 0 & 0 & \sin\theta e^{-ik_x} & \cos\theta e^{-ik_y} \\ 0 & 0 & -\cos\theta e^{ik_y} & \sin\theta e^{ik_x} \\ \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \end{pmatrix}$$

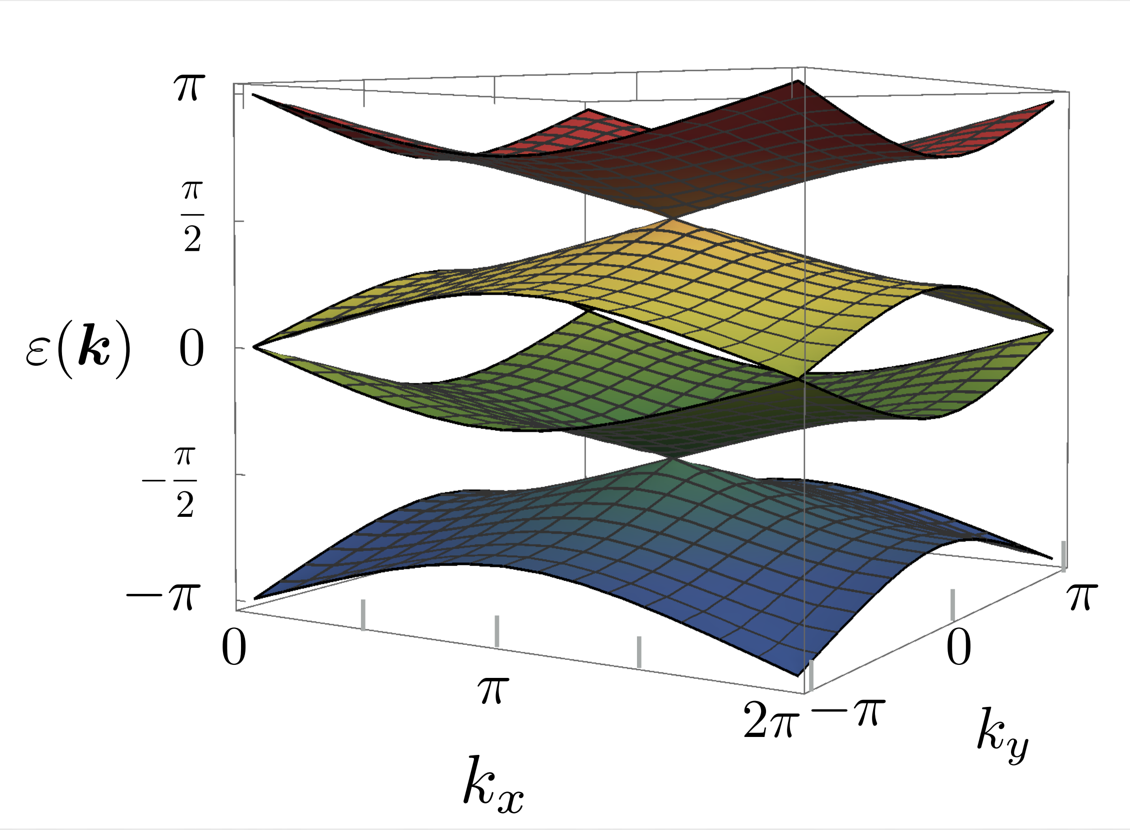


$\theta = \pi/2$ movement around grey squares

$\theta = 0$ movement around white squares

$\theta = \pi/4$ delocalized trajectories

Delplace et al, (2014)-



Bands at $\theta = \pi/4$

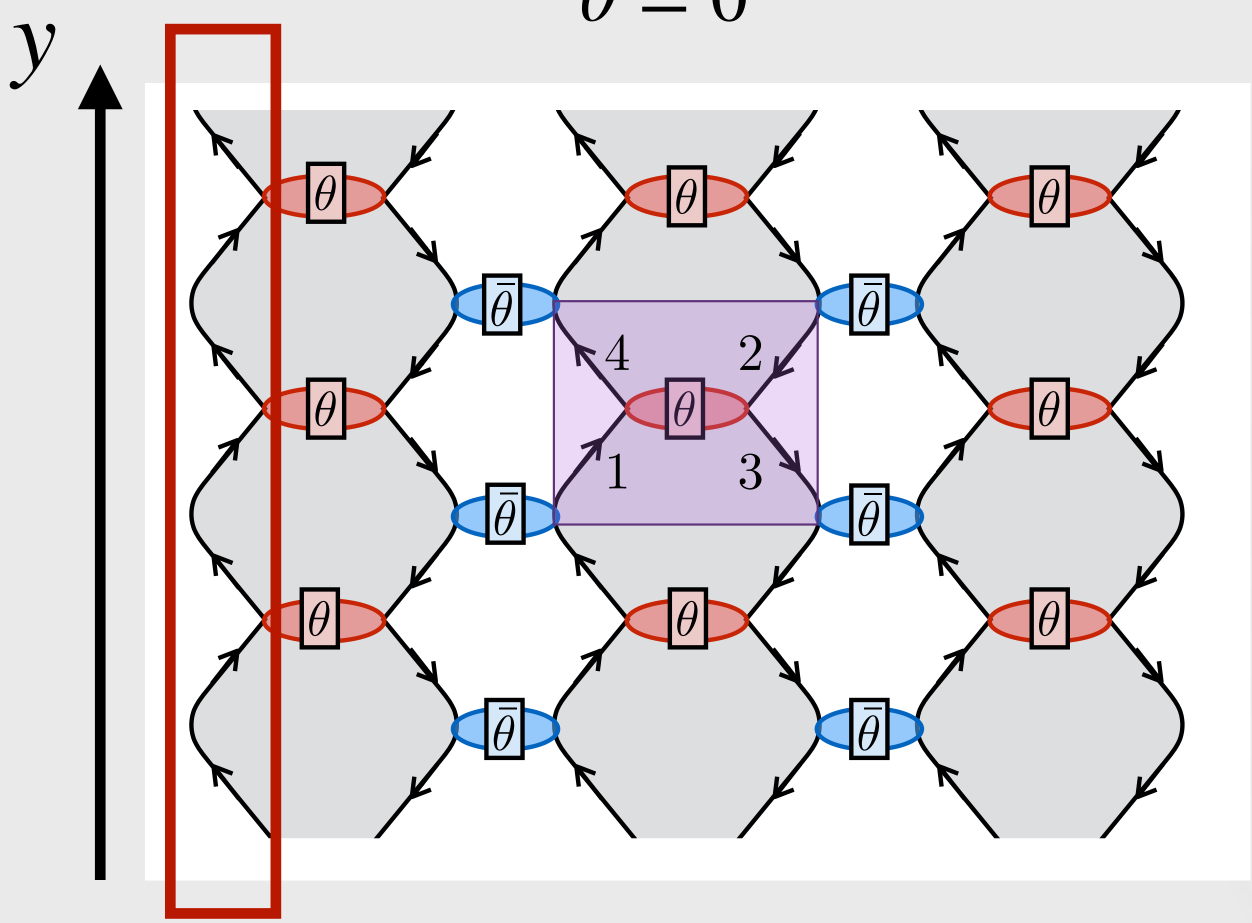
At other values of θ:
four bands separated by gaps.

Each band has Chern number 0.

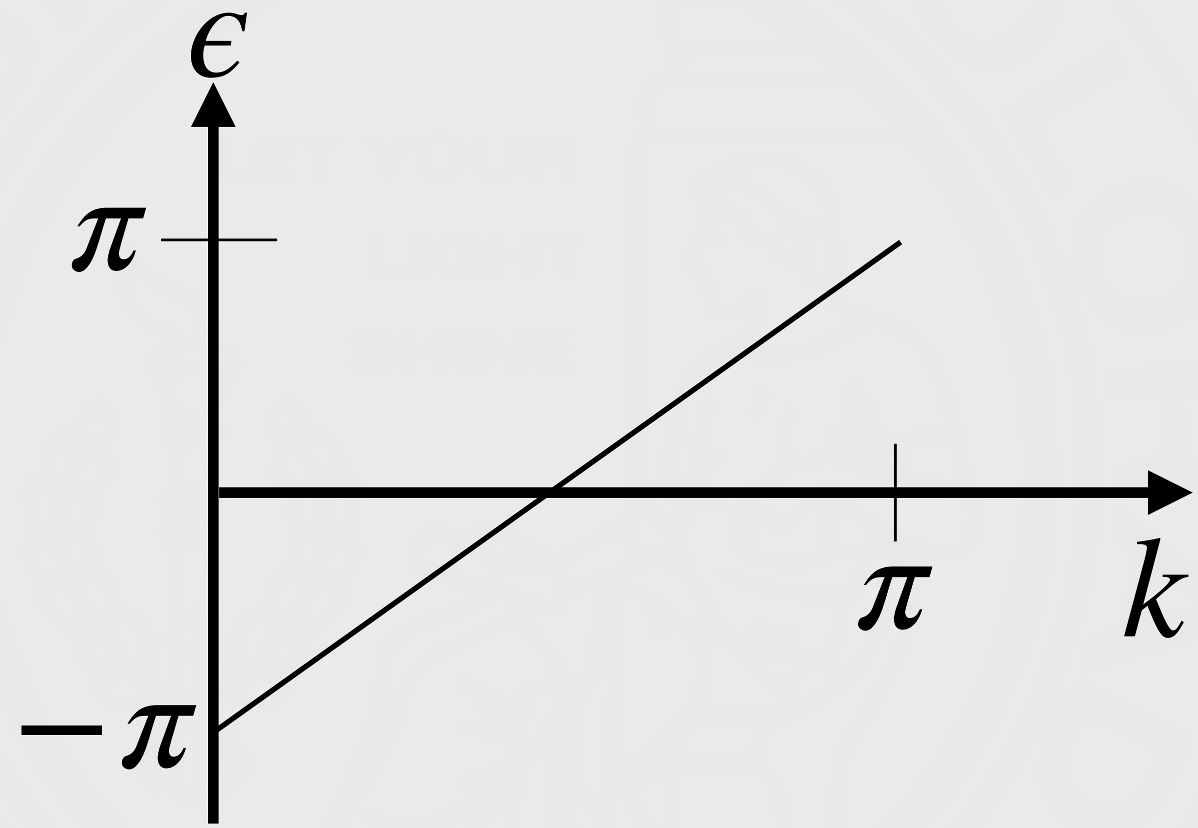
Small deviation from $\theta = \pi/4$
Nonzero Chern density in the vicinity of each Dirac point, canceling between two Dirac points for each band.

Edge states

$\theta = 0$



$\lambda = e^{-ik} = e^{-i\epsilon}$



$\Psi_C = e^{iky}$

Edge states but no Chern numbers?

Chiral Floquet topological phase

Rudner, Lindner, Berg, Levin (2014).

Chiral Floquet Invariant

Edge states but no Chern numbers?

Chiral Floquet topological phase

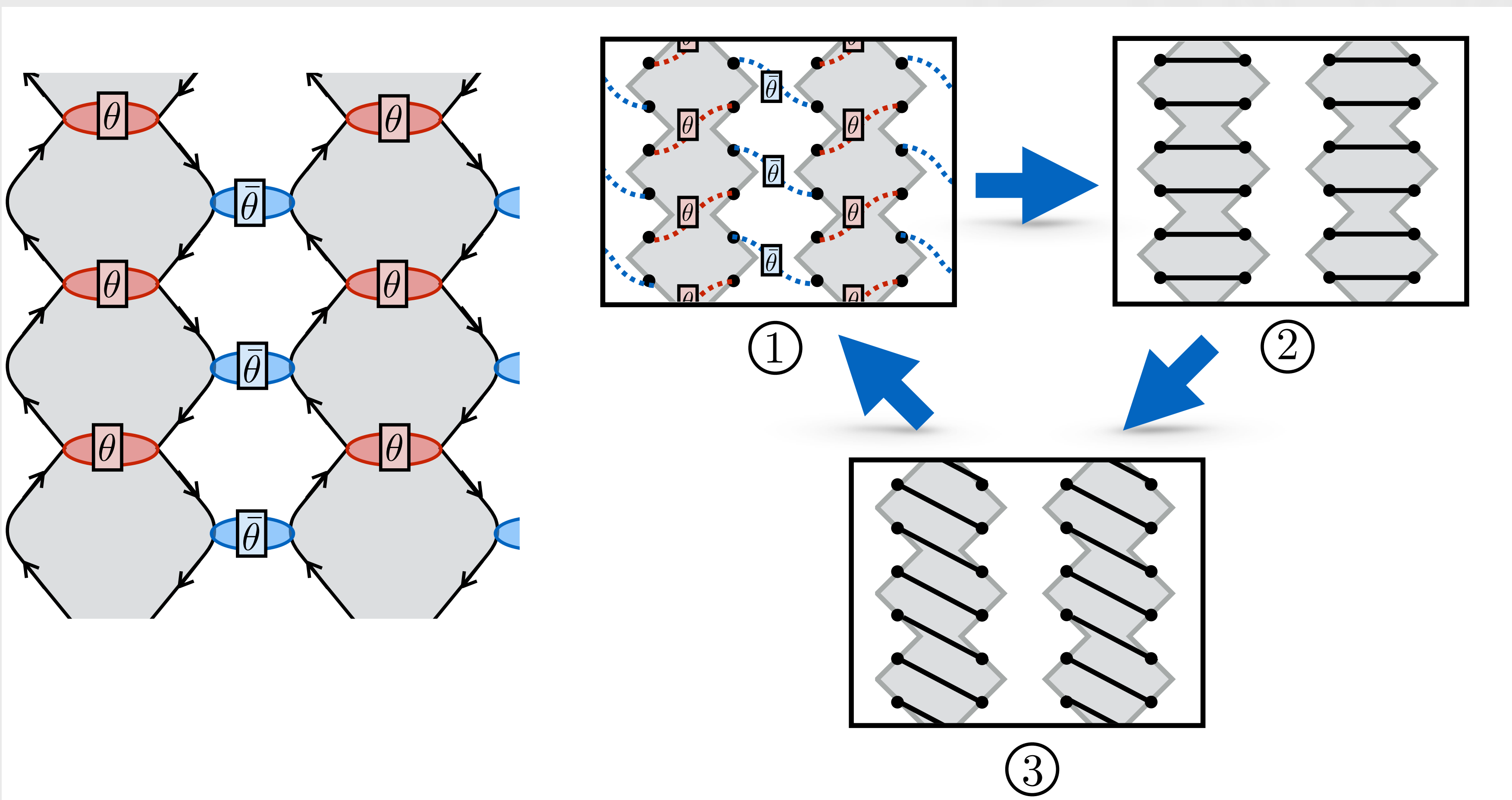
Rudner, Lindner, Berg, Levin (2014).

Chiral Floquet invariant

$$\chi[\tilde{U}] = \frac{1}{8\pi^2}\int dt dk_x dk_y \mathrm{Tr}\left(\tilde{U}^\dagger \partial_t \tilde{U}\left[\tilde{U}^\dagger \partial_{k_x}\tilde{U}, \tilde{U}^\dagger \partial_{k_y}\tilde{U}\right]\right)$$

requires the concept of the time dependent evolution operator

$$U(t)$$



$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\text{out}} = \exp\left(-it\sigma^y\right)\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\text{in}}$$

$t = \pi/2 \ \rightarrow$ exchange

Explicit calculation shows

$$\chi = 0,\ \theta < \pi/4$$
$$\chi = 1,\ \theta > \pi/4$$

Floquet Topological Insulators

Periodically driven systems

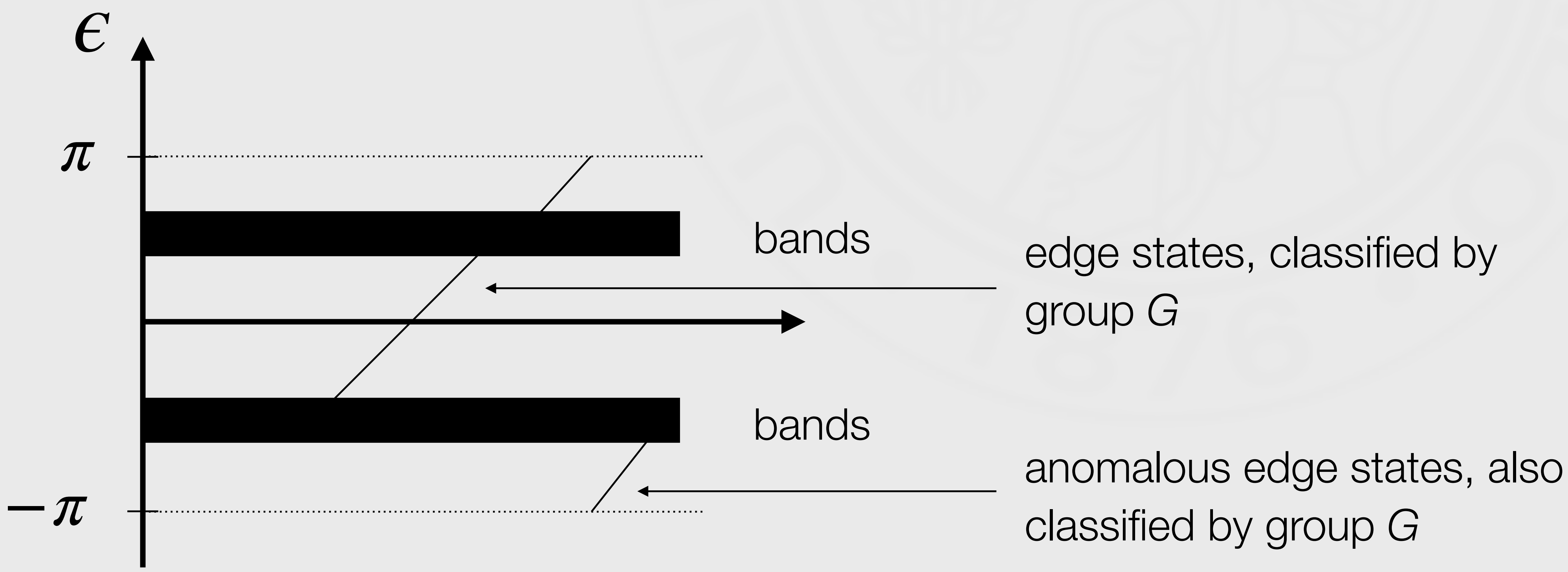
$$i\frac{\partial \psi}{\partial t} = H(t)\,\psi$$

$$H(t+T) = H(t)$$

define evolution operator

$$\psi(t+T) = U(T)\,\psi(t)$$

with eigenvalues $\lambda_n = e^{-i\epsilon_n}$



Roy, Harper (2017): Floquet topological insulators are classified by the group $G \times G$

Paradox

Understood as a stroboscopically evolving Floquet system, network model has two phases

1. Trivial phase
2. Chiral Floquet phase

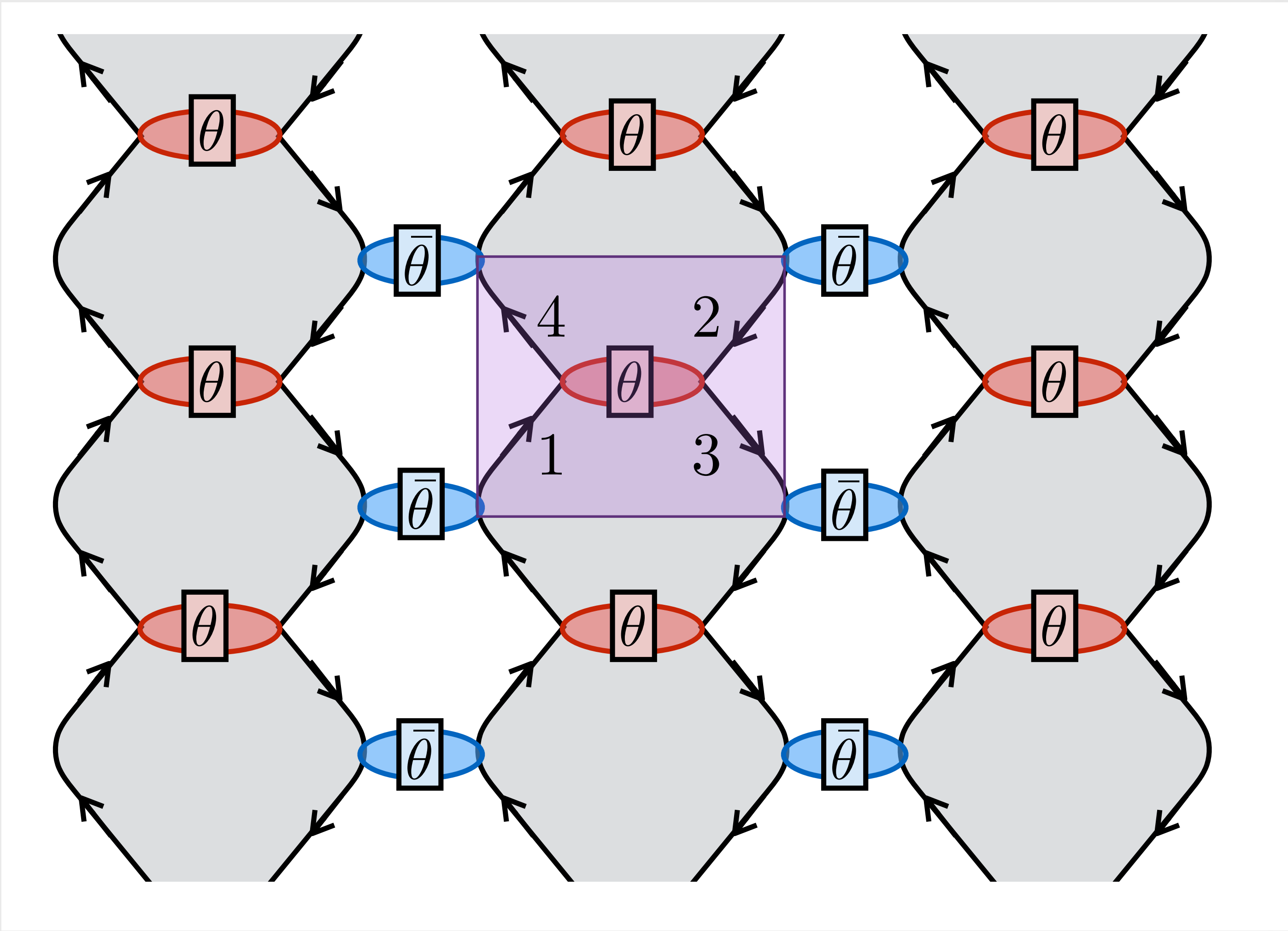
Both have Chern number zero.

Two options:

~~1. Network model describes Chiral Floquet Phases and does not describe Integer Hall Effect with its nonzero Chern number. Different critical exponents for the IQHE plateau transition and network model?~~

2. Network model does describe Integer Quantum Hal Effect. Same critical exponent for IQHE plateau transition. If so, where is the Chern number?

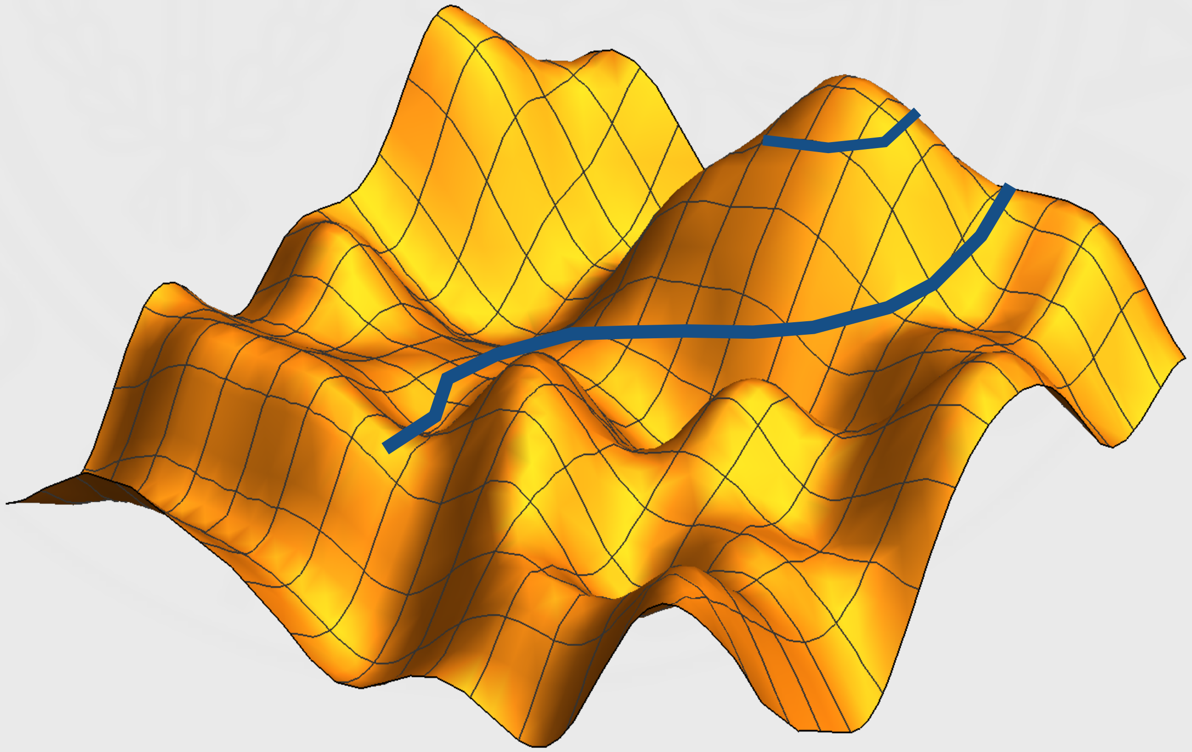
Reinterpretation of the network model



$$H = \frac{1}{2m}\left[\left(-i\frac{\partial}{\partial x} + \frac{eB}{c}y\right)^2 - \frac{\partial^2}{\partial y^2}\right]\psi + V\psi = E\psi$$

$$ma_i = -\frac{\partial V}{\partial x_i} + \frac{eB}{c}\epsilon_{ij}v_j$$

Movement along equipotential lines



We expect θ to be energy dependent, sweeping from 0 to $\pi/2$.

Chern band reconstruction

How can a chiral Floquet topological insulator describe transitions in IQHE?

Chern bands reemerge if energy dependence of θ is taken into account.

θ varies across the Chern bands.

Phase which accumulates across a plaquette should also be given energy dependence.

Hamiltonian eigenstates: eigenstates of the evolution operator, with the eigenvalue 1.

Chern band reconstruction: solve for E

$$\varphi(E) = \epsilon_n(\mathbf{k}, \theta(E)) + 2\pi m$$

Eigenvalues of the evolution operator U

$$\lambda_n = \exp[-i\epsilon_n]$$

For example, we take:

$$\theta(E) = \frac{\pi}{4}\left(\tanh\left[\frac{E + \pi/4}{4\pi}\right] + 1\right)$$

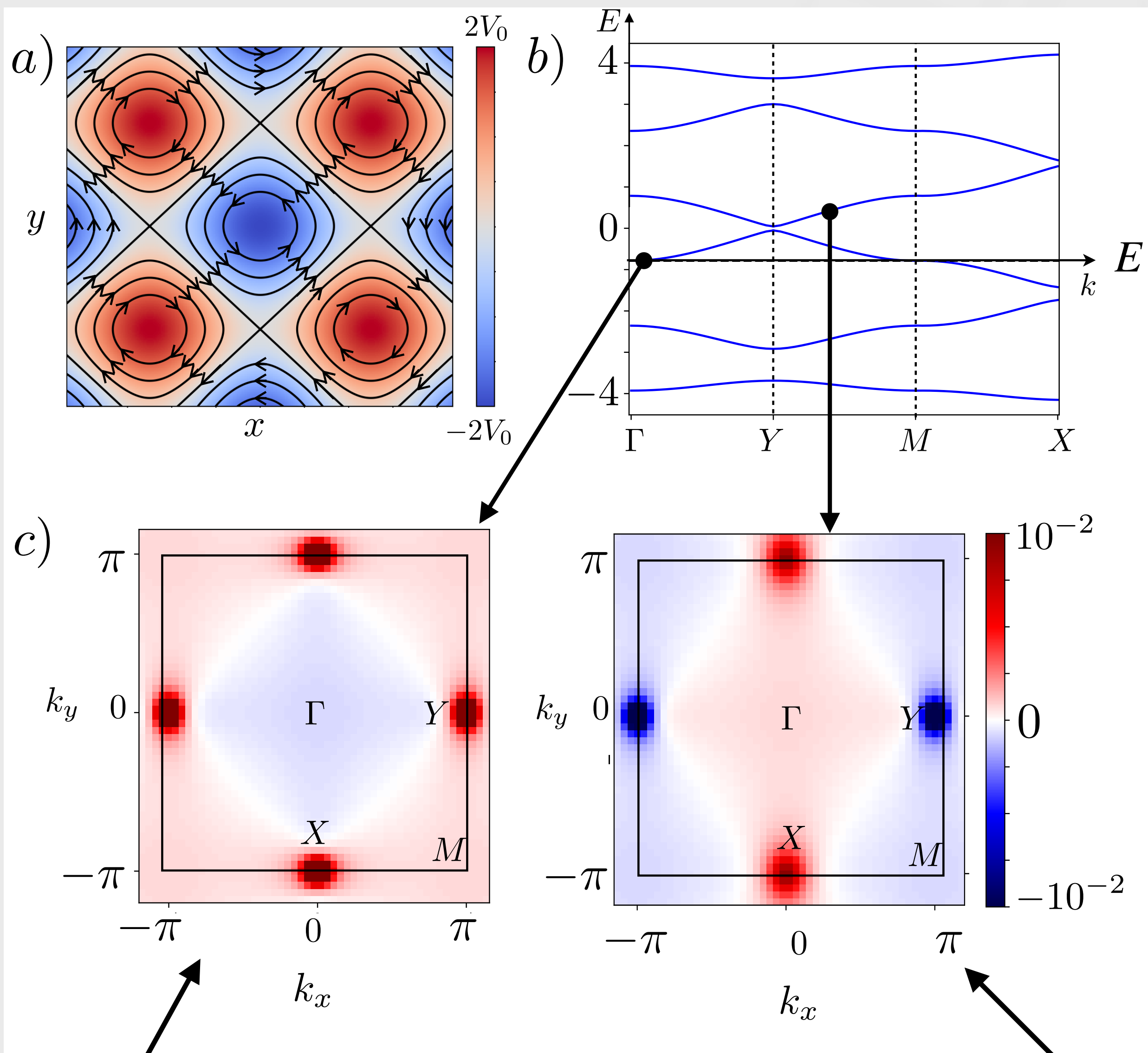
$$\varphi(E) = E$$

$\longrightarrow$ $E_{nm}(\mathbf{k})$ This is the reconstructed band

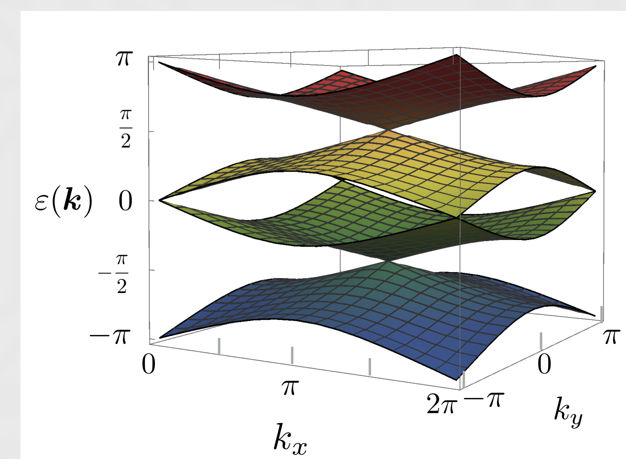
Reconstructed Chern bands

Some of the bands thus found are shown here.

$$\theta(E) = \frac{\pi}{4}\left(\tanh\left[\frac{E+\pi/4}{4\pi}\right]+1\right)$$



$E = -\pi/4 \;\rightarrow\; \theta = \pi/4$



Band structure at $\theta = \pi/4$ for reference

Chern density for two bands.

This band has Chern number 1.

This band has Chern number 0.

Generalizations

Other network models constructed via interpreting scattering as a stroboscopic Floquet evolution provide a similar map between conventional and Floquet topological insulators.

Interacting Floquet models very similar in spirit to the original network model were constructed and used to classify interacting Floquet SPT-like phases. Any relation to non-driven systems?

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Chiral Floquet Phases of Many-Body Localized Bosons

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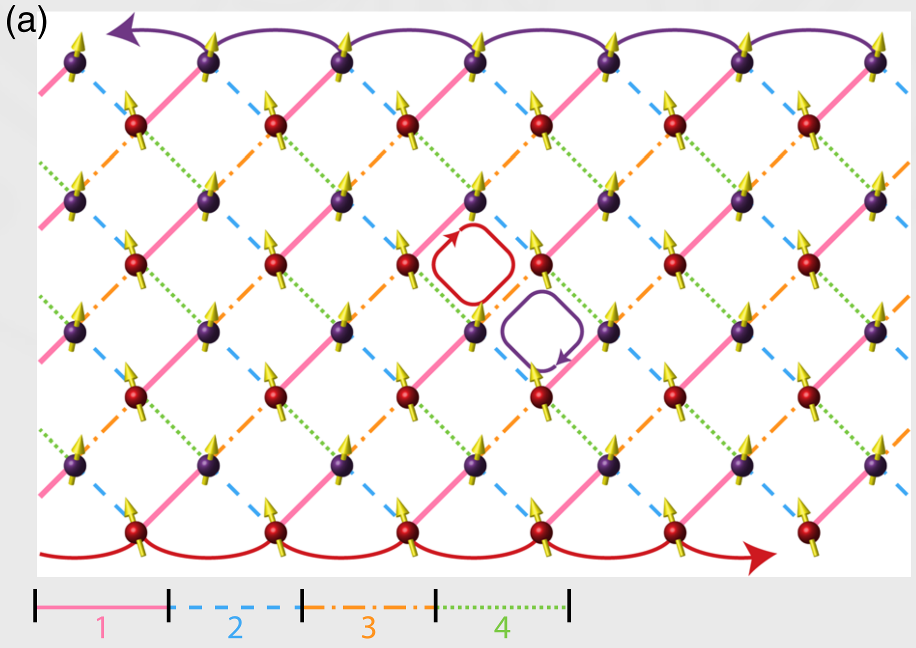

Conclusions & Outlook

- **Chalker-Coddington network model** is both a **chiral Floquet topological insulator** and a **Chern insulator.**

- Provides a **map** between these seemingly different systems

- Shows that disordered Floquet insulators and time-independent topological insulators should have topological class changing transition in **the same universality class.**

- works for **any class** of topological insulators, not only class A (IQHE). Shows how to reconstruct bands for any time-independent systems which was mapped into a stroboscopic Floquet via interpreting scattering as a Floquet evolution.

- **Interactions?** Perhaps yes in 1D, no in 2D and above.