Wormholes and black hole information problems

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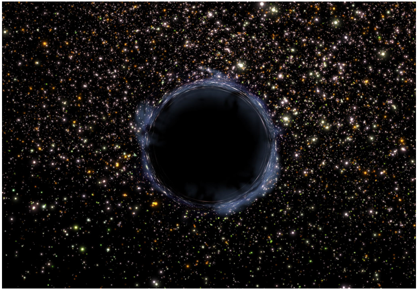
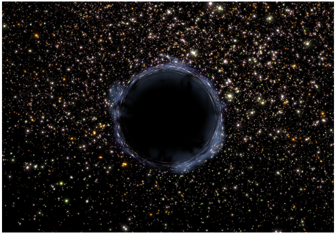
Black holes are Ultra Quantum Matter



- Gauge/gravity duality – black holes (at least outside the horizon) are just like (dual to) ordinary quantum systems.
- Einstein gravity (in the bulk) is a good description only when the quantum system (on the boundary) is strongly coupled – very quantum mechanical.
- Black holes are the Ultimate in Ultra Quantum Matter!

Black hole information problem(s)



- But the bulk geometrical description of black holes, and in particular the existence of smooth horizons, seems to lead to quantum incoherent effects where quantum information is lost.
- This is incompatible with unitary quantum evolution.
- How can this tension be resolved within the bulk description?
- "Black hole information problem(s)."

BHIP II: Correlation functions at long time

- Out of time order:
  Black hole information problem II
  [Maldacena, 2001].
- A simple observable – the two point thermal correlator. In the boundary QM system it can be written

$$\langle O(t)O(0)\rangle = \sum_{mn} |\langle m|O|n\rangle|^2 e^{-\beta E_m} e^{i(E_m - E_n)t}.$$

- This decays in time, at least for a while – the system thermalizes.
- In the bulk this describes injecting a quantum which falls toward the black hole, and at a later time measuring the amplitude for it to stay outside the horizon.
- In classical gravity this decays forever – quasinormal modes
  [Horowitz-Hubeny].

BHIP II: Correlation functions at long time, contd.

- But a finite entropy system, like a black hole, has a discrete spectrum. The correlator must stop decaying at some point. Eventually it oscillates erratically around a small mean value.

$$\langle O(t)O(0)\rangle = \sum_{mn} |\langle m|O|n\rangle|^2 e^{-\beta E_m} e^{i(E_m - E_n)t}.$$

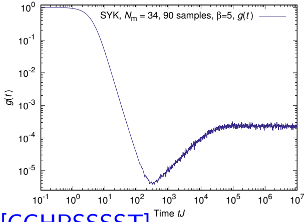
- By ETH matrix element magnitudes vary smoothly. The main actors are the oscillating phases.
- Drop the matrix elements to get the spectral form factor (SFF) (after shifting and rescaling):

$$\sum_{mn} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z(\beta+it)Z(\beta-it).$$

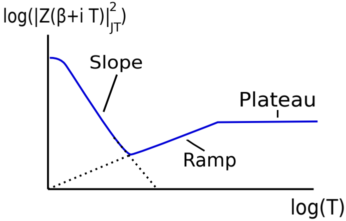
- This also decays and then oscillates.
- What is the gravitational explanation for the end of the decay?

Ensembles of quantum systems – the SFF

- A simpler question, motivated by SYK: Describe the average behavior of the SFF. Consider an **ensemble** of unitary finite entropy quantum systems. Decay still stops.
- Compute the averaged SFF in SYK, $\langle Z(\beta + it)Z(\beta - it)\rangle$.



[CGHPSSSST]



- The slope is the analog of short time decay in the correlator. Eventually the decay turns into a ramp and plateau. Universal pattern in random matrix theory and so in quantum chaotic systems.
- In this talk we will focus on the ramp.

Random matrix statistics

- Averaged (microcanonical) SFF given by

$$\int dEdE' \langle \rho(E)\rho(E') \rangle e^{i(E-E')t}.$$

- $\langle \rho(E)\rho(E') \rangle$ is the pair distribution function of eigenvalues.

$$\langle \rho(E)\rho(E') \rangle \sim \langle \rho(E) \rangle \langle \rho(E') \rangle - \frac{1}{2(\pi(E-E'))^2)} + \mathit{oscillating}$$

- $\langle \rho(E) \rangle \langle \rho(E') \rangle$ is responsible for the slope. Of order e^{2S} at short time.
- $\frac{-1}{(E-E')^2}$ describes long range spectral rigidity. Universal in RMT. Smaller than the slope by e^{-2S}. Responsible for the ramp.

From SYK to RMT

- How do we explain the ramp?
- $SFF = \text{Tr } e^{(-\beta+it)H} \text{ Tr } e^{(-\beta+it)H}$
- In SYK use $H = H_{SYK}$, a $2^{N/2} \times 2^{N/2}$ matrix.
- Very big! A key aspect of many-body quantum chaos. Let $L = 2^{N/2} \sim e^{cN} \sim e^{S}$.
- To understand the ramp, use RMT universality and replace H_{SYK} by an $L \times L$ random matrix (big!). Gaussian Hermitian (GUE) to start.
- Then expand the exponentials in $\langle SFF \rangle$, getting terms like $\langle \text{Tr } H^p \text{ Tr } H^p \rangle$.
- Use 't Hooft double-line diagram perturbation theory in $1/L$.
- Wick contractions with $\langle HH \rangle \sim \frac{1}{L}$.

Disks

H: L×L matrix (random hermitian)

$\langle H H \rangle = \;=\!=\!= \; \times \left(\frac{1}{L}\right)$

Diagrams weighted by

L^{χ}, χ = Euler Character

$\langle \mathrm{tr} H^p \, \mathrm{tr} H^p \rangle$: $(= \cdots > -)$

L^2

$\chi = 1 + 1 = 2$

$= \langle \mathrm{Tr}\, H^p \rangle \langle \mathrm{Tr}\, H^p \rangle$ 2 Disks

Explains the slope.

Cylinder

$\chi = 0$

L^0

Cylinder

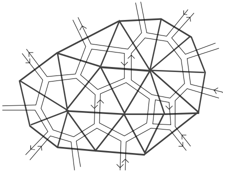
This (plus cyclic rotations) explains the ramp.

From RMT to SYK

- This was easy. Now explain the ramp in SYK [Saad-SS-Stanford].
- Two traces, so two replicas of SYK: L, R. Need collective fields $G_{LL}, G_{LR} \dots$.
- Disks are of order $L^2 \sim e^{2cN}$. Leading, disconnected, saddle point with $G_{LR} = 0$.
- Cylinders are of order L^0. A perturbative correction in RMT, but of relative size e^{-2cN} in SYK. Nonperturbative in the SYK $1/N$ perturbation expansion!
- Due to a subleading, connected, saddle point in SYK with $G_{LR} \neq 0$.

From RMT to quantum gravity

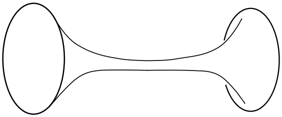
- Now explain in the 2D gravity theory, JT gravity, that is the low energy limit of SYK [Saad-SS-Stanford].
- $G_N \sim 1/N$ so the ramp will be a nonperturbative, $\sim e^{-1/G_N}$, effect in quantum gravity.
- Lesson from the 1980s – use non-Gaussian matrix ensembles to describe 2D gravity. Interactions "fill in" the geometry.



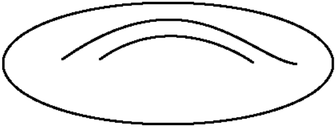
- Replace H_{SYK} by a particular non-Gaussian random matrix ensemble.

From Cylinders to wormholes

- After "filling in," the cylinder diagram becomes the "double trumpet."



- This is an example of a spacetime wormhole (not an Einstein-Rosen bridge).
- This additional contribution to the gravitational path integral explains the ramp in this theory.
- The correlation function is a single trace quantity – one boundary. The geometry responsible for the ramp in this quantity is a handle on the disk [Saad].

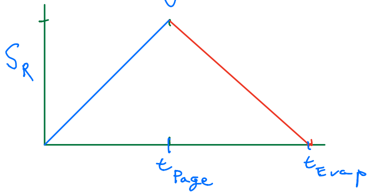


BHIP I

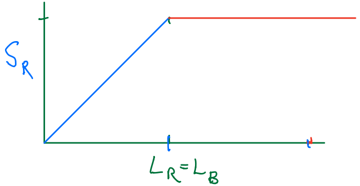
- We now turn to the first black hole information problem, chronologically. It involves evaporating black holes [Hawking, 1975].
- Consider a black hole that evaporates into Hawking radiation (unlike large AdS black holes).
- Assume that the entire system, black hole (B) and radiation (R), form one isolated quantum system. Now trace out the black hole part of the Hilbert space leaving ρ_R, the radiation density matrix.
- At short times the radiation looks thermal, so the (von Neumann) entropy of the radiation $S_R^{vN} = -\mathrm{Tr}\, \rho_R \log \rho_R$ increases monotonically.
- But after the black hole completely evaporates the radiation is the entire system – a pure state. So $S_R^{vN} = 0$. We expect something like the Page curve:

The Page curve I

The Page Curve:

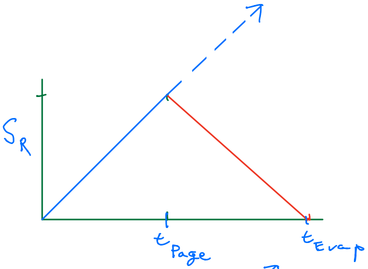


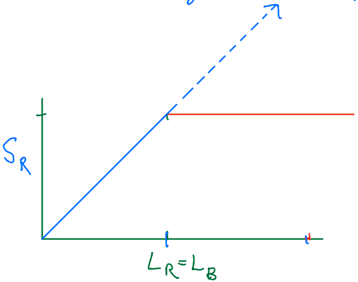
Easier to deal with a (non-evaporating) BH in equilibrium with an external radiation bath



The Page curve II

Hawking's Calculation:





Beyond Hawking

- What did Hawking leave out?
- Again, I'll proceed out of time order.
- The first key insight here was made using AdS/CFT technology – especially quantum corrected Ryu-Takayanagi surfaces [Penington, Almheiri-Engelhardt-Marolf-Maxfield].
- Then, building on some work by [Lewkowycz-Maldacena] it became possible to uncover the underlying gravitational dynamics [Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini; Penington-SS-Stanford-Yang].
- I'll try to give a sketch of this work.

Beyond Hawking, contd.

- Study in a concrete model. Start with two coupled SYKs:
  B represents the black hole, has N_B fermions, $L_B = e^{cN_B}$;
  R represents the radiation, $N_R, L_R = e^{cN_R}$.
- Couple them and let them evolve and entangle [Gu-Lucas-Qi].
- Then trace out B to get ρ_R.
- Because of chaotic evolution the state of the combined system starts to look like a random vector in the combined Hilbert space (ETH).
- So model by a random state [Page, 1993].

Random state model

- Write a state in the combined BR system as

$$|\psi\rangle_{BR} = \sum_{\alpha i} \psi_{i\alpha} |\alpha i\rangle, \ \alpha = 1 \ldots L_B, \ i = 1 \ldots L_R$$

- $\psi_{i\alpha}$ are independent complex gaussian variables with variance of magnitude $\frac{1}{L_B L_R}$. (This ensures state normalization at large L_B, L_R).
- $\psi_{i\alpha}$ is a complex rectangular random matrix. This is the (complex) Wishart ensemble. Random matrices again...
- Compute the radiation density matrix: $(\rho_R)_{ij} = \psi^*_{i\alpha} \psi_{\alpha j}$.
- Now compute entropies. The vN entropy is a a bit tricky. Instead start with the (second) Renyi entropy. Use two "replicas."
- Compute the (ensemble averaged) purity $\langle \mathrm{Tr}\, \rho^2 \rangle$ and then $\langle S_R^{(2)} \rangle = \langle - \log \mathrm{Tr}\, \rho_R^2 \rangle$.
- Use 't Hooft double line perturbation theory again.

Random state model, contd.

$\Psi_{\alpha i}$

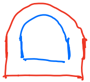
$i = 1 \cdots L_R$

$\alpha = 1 \cdots L_B$

$\rho_{R\, ij} = \Psi^*_{i\alpha}\Psi_{\alpha j} \longrightarrow$ 

$\mathrm{tr}\,\rho_R \longrightarrow$ 

$\langle \Psi^*_{i\alpha}\,\Psi_{\beta j}\rangle =$  $\times \left(\frac{1}{L_B L_R}\right)$

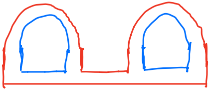
$\langle \mathrm{tr}\,\rho_R \rangle =$ 

$= \frac{L_B L_R}{L_B L_R} = 1$ ✓

Random state model, contd.

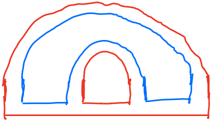
$\mathrm{tr}\,\rho_R^2 \longrightarrow$

$\langle \mathrm{tr}\,\rho_R^2 \rangle =$



(I)

$+$



(II)

$$(\mathrm{I}) = \frac{L_B^2 L_R}{(L_B L_R)^2} = \frac{1}{L_R}$$

$$(\mathrm{II}) = \frac{L_B L_R^2}{(L_B L_R)^2} = \frac{1}{L_B}$$

$$\langle \mathrm{tr}\,\rho_R^2 \rangle = \frac{1}{L_R} + \frac{1}{L_B}$$

Random state model, contd.

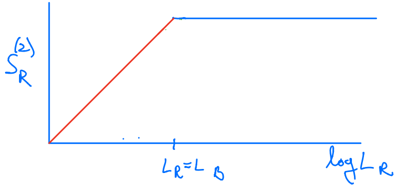
$$\langle \mathrm{tr}\, \rho_R^2 \rangle = \frac{1}{L_R} + \frac{1}{L_B}$$

$$\langle S_R^{(2)} \rangle = -\log \langle \mathrm{tr}\, \rho_R^2 \rangle \quad \text{(self-averaging)}$$

$$= \log \left(\frac{L_R L_B}{L_R + L_B} \right)$$

$$L_R \ll L_B, \quad = \log L_R - O\left(\frac{L_R}{L_B}\right)$$

$$L_R \gg L_B, \quad = \log L_B - O\left(L_B / L_R\right)$$



Page curve

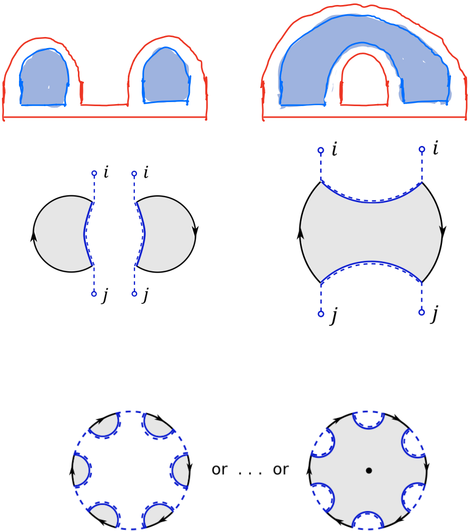
From random states to SYK

- $\langle \text{Tr}\ \rho_R^2 \rangle = 1/L_R + +1/L_B$. When $L_R \ll L_B$ the second term is a perturbative correction in RMT.
- Now return to the SYK context where $L_B \sim e^{cN_B},\ L_R \sim e^{cN_R}$.
- Now $\langle \text{Tr}\ \rho_R^2 \rangle = e^{-cN_R} + e^{-cN_B}$. When $N_R \ll N_B$ the second term is a nonperturbatively small correction in SYK ($\frac{1}{N}$) perturbation theory ($\frac{1}{N} \sim G_N$).
- Due to another subleading saddle point in the SYK collective field integral, with $G_{BR} \neq 0$.
- As N_R grows the contribution of the main saddle decreases, making it natural for the subleading saddle to become dominant at some point.
- This is less obvious in $S_R^{(2)} \sim cN_R + O(e^{-N_B})$ where the contribution of the main saddle grows.

From random states to gravity, contd.

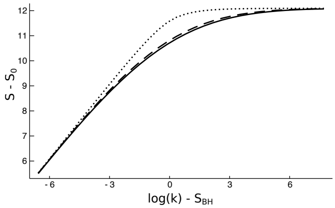
- Now go to gravity. Introduce a non-Gaussian weight for the H acting in the B Hilbert space.
- ρ_R becomes (schematically) $(\rho_R)_{ij} = \psi^*_{i\alpha}\, (e^{-\beta H})_{\alpha\beta}\, \psi_{\beta j}$. The H diagrams " fill in" the gravitational geometry, as before.
- This gravitational "filling in" is important for understanding finite temperature (as opposed to fixed energy) effects.

From random states to gravity, contd.



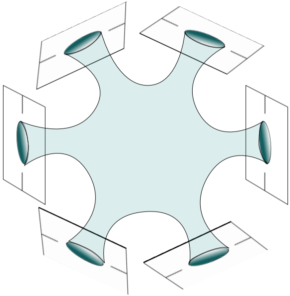
von Neumann entropy from gravity

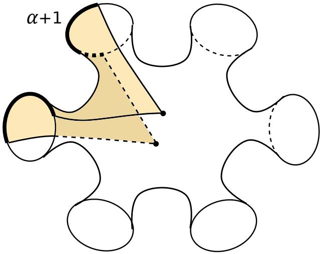
- Can sum over all (planar) diagrams for $\langle \mathrm{Tr}\ \rho_R^n \rangle$ including non $\mathbb{Z}_n$ symmetric ones.
- Then analytically continue $n \to 1$ to get S_R^{vN}.



- The rounding of the transition is a finite temperature effect.
- Then one can get more elaborate and model the radiation as a 1+1 D CFT.

Replica wormhole pinwheels



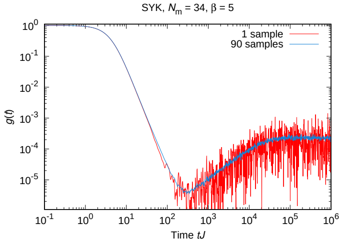


[Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini; Penington-SS-Stanford-Yang]

What did Hawking leave out? Wormholes.

Fluctuations

- But it is clear that wormholes are only part of the story.
- If you consider a fixed boundary Hamiltonian, like Super Yang Mills, rather than an ensemble, the SFF will look like:



- The Page curve is self averaging, and has small fluctuations. But other quantities, like individual matrix elements $(\rho_R)_{ij}$ will have large fluctuations, and cannot be described by wormholes.
- What needs to be added for a complete bulk description?