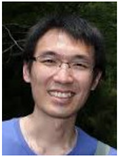
Emergent higher symmetry from topo. order

Xiao-Gang Wen (MIT), Sep., 2019 UQM

arXiv:1812.02517

Simons Collaboration on Ultra-Quantum Matter arXiv:1908.02613 **Lokman Tsui** & XGW



Lattice models that realize Z_n-1-symmetry protected topological states for even n

For the Z_2 case, the bulk 1-SPT orders are given by $\mathbb{Z}_4$. The 1-symmetric boundary can be gapped with double semion topological order for $m = 1$ and toric code for $m = 2$. The bulk ground state wavefunction amplitude is given in terms of the linking numbers of loops.

arXiv:1906.08270 **Michael DeMarco** & XGW

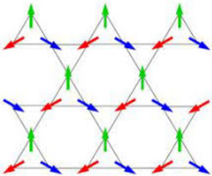
Lattice realization of compact U(1) Chern-Simons theory with exact 1-symmetries

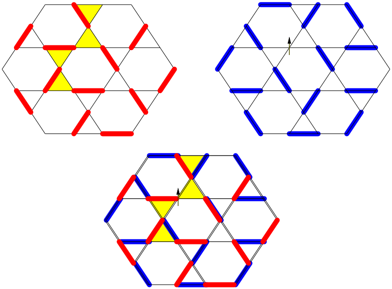
Proposed a bosonic $U(1)$ rotor model on a three dimensional space-time lattice which realizes a Chern-Simons field theory $S = \int \mathrm{d}^3 x \, \frac{K_{IJ}}{4\pi} A_I \mathrm{d} A_J$ at low energies. The lattice model has the exact (anomalous) 1-symmetries that appears in the Chern-Simons field theory.

Topological excitations in Z_2 spin liquid (herbertsmithite)

Mei Chen He Wen, arXiv::160609639

Z_2-spin liquid in $ZnCu_3(OH)_6Cl_2$





- **Topological excitations**:

(1) e = spin-1/2 boson. (2) f = spin-1/2 fermion.

(3) m = spin-0 boson. (4) 1 = spin-0 boson.

Read Sachdev, PRL **66** 1773 (91); Wen, PRB **44** 2664 (91).

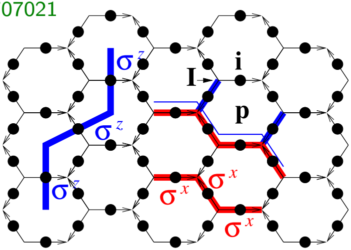
- If m has low energy, what kind of phase it can induce?
- If e has low energy, what kind of phase it can induce?
- If f has low energy, what kind of phase it can induce?

Can we get a $SO(3)$ symmetric phase with trivial topological order?

Can we get a $SO(3)$ symm.-broken phase with trivial topo. order?

Exactly soluble model and string operator

- Toric code model: Kitaev quant-ph/9707021
  $H = -U\sum_{\boldsymbol{I}} \hat{Q}_{\boldsymbol{I}} - g\sum_{\boldsymbol{p}} \hat{F}_{\boldsymbol{p}}$
  $\hat{Q}_{\boldsymbol{I}} = \otimes_{\text{legs of } \boldsymbol{I}} \sigma^z_{\boldsymbol{i}}$,
  $\hat{F}_{\boldsymbol{p}} = \otimes_{\text{edges of } \boldsymbol{p}} \sigma^x_{\boldsymbol{i}}$
- Topological excitations:
  e-type: $\hat{Q}_{\boldsymbol{I}} = 1 \to \hat{Q}_{\boldsymbol{I}} = -1$
  m-type: $\hat{F}_{\boldsymbol{p}} = 1 \to \hat{F}_{\boldsymbol{p}} = -1$



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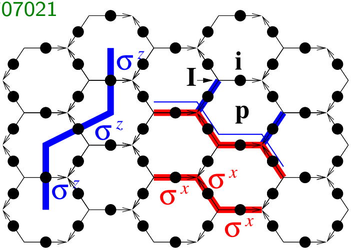


- Type-e string operator $W_e(C_1) = \otimes_{i \in C_1} \sigma_i^x$
- Type-m string operator $W_m(\tilde{C}_1) = \otimes_{i \in \tilde{C}_1} \sigma_i^z$
- Type-f string operator $W_f(C_1 \otimes \tilde{C}_1) = \otimes_{i \in_1 C_1} \sigma_i^x \otimes_{i \in \tilde{C}_1} \sigma_i^z$

- $[H, W_e^{\text{closed}}] = [H, W_m^{\text{closed}}] = [H, W_f^{\text{closed}}] = 0$
  $\to$ Closed strings cost no energy
- $[\hat{Q}_I, W_e^{\text{open}}] \neq 0$ flip $\hat{Q}_I \to -\hat{Q}_I$, $[\hat{F}_p, W_m^{\text{open}}] \neq 0$ flip $\hat{F}_p \to -\hat{F}_p$
  $\to$ **open-string create a pair of topo. excitations at their ends**

Global symmetry (0-symmetry) and higher symmetry

- The toric model has a global $\mathbb{Z}_2$ symmetry

$$U_g H = H U_g, \quad U_g = \otimes_{\boldsymbol{i}} (\sigma^z_{\boldsymbol{i}})^g, \quad g = 0, 1 \in \mathbb{Z}_2$$

\- U_g acts on the whole space *ie* the codimension-0 subspace.
$\to$ U_g generates a 0-symmetry.

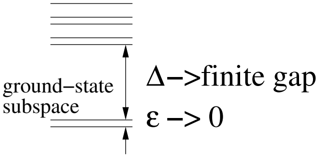
- The toric model has two $\mathbb{Z}_2$ higher symmetry generated by

$$W_e(C_1)H = HW_e(C_1) \qquad W_m(\tilde{C}_1)H = HW_m(\tilde{C}_1),$$
$$W_e^2(C_1) = 1, \qquad W_m^2(\tilde{C}_1) = 1$$

\- C_1 and $\tilde{C}_1$ are **any closed subspaces** of codimension-1
$\to$ $W_e(C_1)$ and $W_t(\tilde{C}_1)$ generates two 1-symmetries.

- Those strings (higher symmetries) were studied by Kiteav quant-ph/9707021; Levin-Wen cond-mat/0302460; Nussinov-Ortiz cond-mat/0605316, as **(low-dim.) gauge symm.** or **logic operators** Gaiotto-Kapustin-Seiberg arXiv:1412.5148 introduced **higher form symmetry** and view them as generalized global symmetry.

Ground state degeneracy and 0-symmetry breaking

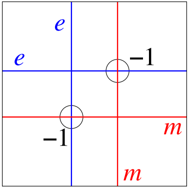


- Transverse field Ising model:
  $H = -\sum(J\sigma_i^z\sigma_{i+\delta}^z + h\sigma_i^x), \quad |h| < J$
- The 0-symmetry is generated by
  $U = \prod_i \sigma_i^x$: $U|\uparrow\uparrow\cdots\rangle = |\downarrow\downarrow\cdots\rangle$
- Spontaneous 0-symmetry breaking:

  Ground states transform non-trivially $U|\uparrow\uparrow\cdots\rangle = |\downarrow\downarrow\cdots\rangle$
  or $|\pm\rangle = |\uparrow\uparrow\cdots\rangle \pm |\downarrow\downarrow\cdots\rangle$ $U|+\rangle = |+\rangle, \quad U|-\rangle = -|-\rangle.$

  Spontaneous symmetry breaking = U is not proportional to an identity operator in the degenerate ground state subspace.

Ground state degeneracy and higher symmetry breaking



- When strings cross,
  $W_e(C_1)W_m(\tilde C_1) = (-)^{\#\text{ of cross}} W_m(\tilde C_1)W_e(C_1)$
  $\to$ 4-fold degeneracy on torus
  4^g-fold degeneracy on genus g surface
- The degeneracy is **topologically robust**
  Degeneracy remain exact for any perturbations localized in a finite region. Degeneracy remain exponentially small for any perturbations.
  $\to$ **The toric code model realizes a Z_2 topological order.**
- The ground states in 4^g-dimension subspace transform non-trivially under the 1-symmetry transformation, *ie* $W_e(C_1)$, $W_m(\tilde C_1)$, and $W_f(C_1 \otimes \tilde C_1)$ are not proportional to identity in the degenerate ground state subspace. $\to$

  The Z_2-1-symmetries generated by $W_e(C_1)$, $W_m(\tilde C_1)$, and $W_f(C_1 \otimes \tilde C_1)$ are all spontaneously broken

Topo. order and spontaneous higher symmetry breaking

- Robust ground state degeneracy on space of non-trivial homotopy type defines **topological order** Wen PRB **40** 7387 (89)

- String/membrane operators act non-trivially on those degenerate ground states → spontaneous higher symmetry breaking.

- **Any spontaneous discrete higher symmetry breaking → topological order in 2+1D and higher**.
  But not all topo. order can be viewed spontaneous higher symmetry breaking, such as those described by discrete non-Abelian gauge theory

- Levin-Wen cond-mat/0302460; Hastings-Wen cond-mat/0503554
  string condensation → topological order.
- Nussinov-Ortiz cond-mat/0605316
  low-dimensional gauge symmetry → topological order.
- Gaiotto-Kapustin-Seiberg arXiv:1412.5148
  spontaneous breaking of higher form symmetry → topo. order.

The robustness of emergent higher symmetry

- 0-symmetries (global symmetry) appear naturally in condensed matter systems.
  But higher symmetry only appear in some fine tuned toy model on lattice.

\- For example, in our toric-code example, if we add perturbations, $H' = H + \delta H$ no longer commute wuth the 1-symmetry transformation $W(C_1)$. There is no higher symmetry.

Is higher symmetry useful in condensed matter?

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Is higher symmetry useful in condensed matter?

- Hastings-Wen cond-mat/0503554 showed that we can always find a fattened string operator
  $$W^{fat}(C_1) = U_{LU} W(C_1) U_{LU}^\dagger$$
  such that H' and $W^{fat}(C_1)$ commute in a low energy subspace (*ie* ground state subspace).

Emergent higher symmetries can appear naturally in topologically ordered phases. In this sense, higher symmetry is useful in condensed matter.

The topo. robustness of emergent gapless gauge bosons

Spontaneous broken higher symmetry $\to$ "topological order":

- Spontaneous broken finite higher symmetry $\to$ topological order in 2+1D and higher.

Levin-Wen cond-mat/0302460; Nussinov-Ortiz cond-mat/0605316

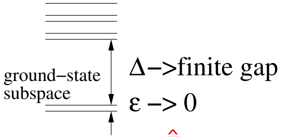
- Spontaneous broken $U(1)$ higher symmetry $\to$ gapless gauge bosons. Gaiotto-Kapustin-Seiberg arXiv:1412.5148

- Spontaneous broken continuous 0-symmetry $\to$ gapless Goldstone bosons

- **The emergent $U(1)$ higher symmetry at low energies is exact and the emergent gapless photon is robust against all the perturbations for 3+1D and higher.** Hastings-Wen cond-mat/0503554
- In contrast, the emergent 0-symmetry at low energies may not be exact and the gapless Goldstone bosons may not be robust against some perturbations.

Emergent higher symm. in the presence of topo. excitation

The above emergent higher symmetry appears below the minimal gap Δ.



- Now consider the model with $U \gg g, J$

$$H = -U \sum_{l} \hat{Q}_{l} - g \sum_{p} \hat{F}_{p} - J \sum_{i} \sigma_{i}^{z}, \quad \hat{Q}_{l} = \otimes_{i \in l} \sigma_{i}^{z}, \ \hat{F}_{p} = \otimes_{i \in p} \sigma_{i}^{x}$$

- As we go from $g \gg J$ to $g \ll J$ the ground state undergoes a Z_2 topological order to trivial product state phase transition,

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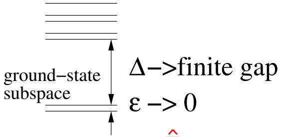

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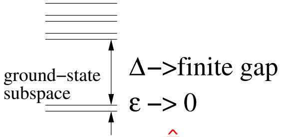

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- The above Hamiltonian and the transition has the Z_2^m-1-symmetry $W_m(\tilde{C}_1) = \otimes_{i \in \tilde{C}_1} \sigma_i^z$, but does not have the Z_2^e-1-symmetry $W_e(C_1) = \otimes_{i \in C_1} \sigma_i^x$ and the Z_2^f-1-symmetry $W_f(C_1 \otimes \tilde{C}_1)$.

- The end of W_m (the 1-symmetry generator) is the m particle. The low energy allowed excitations of the above Hamiltoniam are the particles with trivial mutual statistics with the m particle. $\to$ The low energy allowed excitations include the m, but not include the e and the f (the fermions) which have a large gap U.

Emergent higher symm. in the presence of topo. excitation

- The Z_2 topo. order in 2+1D has three type of topo. excitations:
  The Z_2-charge e – boson
  The Z_2-flux m – boson
  The charge-flux bound state $f = m \otimes e$ – fermion
  The three particles have mutual π statistics respect to each other.

- Below the minimal gap of the three particles $\Delta_e, \Delta_m, \Delta_f$, we have three Z_2-1-symmetries generated by closed string operators $W_e(C_1)$, $W_m(\tilde{C}_1)$, and $W_f(C_1 \otimes \tilde{C}_1)$.

- If $\Delta_m \ll \Delta_e, \Delta_f$, then below Δ_e, Δ_f (but may be above Δ_m), we have a Z_2-1-symmetry generated by closed string operators $W_m(\tilde{C}_1)$, but not the ones from $W_e(C_1)$ and $W_f(C_1 \otimes \tilde{C}_1)$.
  Low energy allowed particles are $\mathcal{C}_a = \{m\}$. The 1-symmetry is generated by string operators for the particles $\mathcal{C}_t = \{m\}$.
- If we reduce Δ_m to make it negative, we will induce a Bose-condensation of the Z_2-flux and a Z_2^m-1-symmetric confinement transition: the Z_2 topo. order $\to$ trivial product state.

Emergent higher symm. in the presence of topo. excitation

- If $\Delta_f \ll \Delta_e, \Delta_m$, then below Δ_e, Δ_m (but may be above Δ_f), we have a Z_2-1-symmetry generated by closed string operators $W_f(C_1 \otimes \tilde{C}_1)$, but not the ones from $W_e(C_1)$ and $W_m(\tilde{C}_1)$.

  Low energy allowed particles $\mathcal{C}_a = \{f\}$, The 1-symmetry is generated by string operators for the particles $\mathcal{C}_t = \{f\}$.

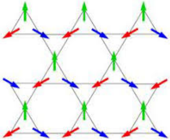
- *If we reduce Δ_f to make it negative, can we still induce confinement transition to change the Z_2 topological order to a trivial product state with no topological order?*

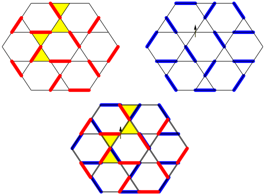
- f is a fermion, and cannot Bose-condense. But it can condense into some other topologically ordered state. Can the new topological order cancel the parent Z_2 topological order to produce a trivial phase without topological order?

  The condensation of f is a Z_2^f-1-symmetric phase transition. But the Z_2^f-1-symmetry is **anomalous**, and a Z_2^f-1-symmetric phase cannot be a trivial product state.

Phase transitions in Z_2 spin liquid induced by topological excitations

Spin liquid in $ZnCu_3(OH)_6Cl_2$





- Proliferation of spin-0 boson m to form a condensed state can induce a $SO(3)$ symmetric phase with trivial topological order.

- Proliferation of spin-1/2 boson e to form a condensed state can induce a $SO(3)$ symmetry-broken phase with trivial topo. order.

- Proliferation of spin-1/2 fermion f to form any kind of states can never induce a phase with trivial topological order regardless of $SO(3)$ symmetry, since the emergent Z_2-1-symmetry is anomalous.