Gapless topological states: an overview

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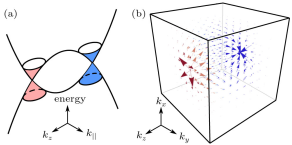

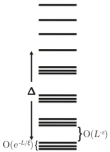

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What is a gapless topological state?

This term has all the ambiguities of 'topological state' and more!

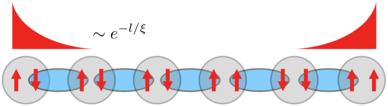
Generous definition of 'topological state': a state of matter which cannot be characterized by a local order parameter.

Outline: possible interpretations of the title.

1. Gapless states with a boundary anomaly
2. Gapless states with fractional quasiparticles
3. Gapless states with a bulk anomaly
4. Gapless states with topological order

Definition 1: States with a boundary anomaly

- Gapless SPTs
- Critical points of SPTs



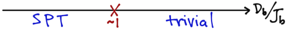
Gapless SPTs

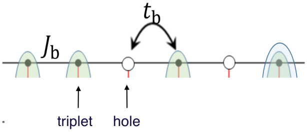
Surely incomplete references: [Cheng-Tu 2011, Grover-Vishwanath 2012, Keselman-Berg, Baum-Posske-Fulga-Trauzettel-Stern, Parker-Scaffidi-Vasseur, Ruhman-Altman, Jiang-Li-Seidel-DH Lee, Verresen-Jones-Pollmann-Thorngren, Ji-Shao-XGW 2019]

- Some can be made with free fermions.
  [Verresen-Jones-Pollmann 2017] identify a topological bandstructure index whose nonvanishing guarantees exponentially-localized edge modes.
- Some require interactions.
  Many new kinds of Luttinger liquids.
  [Cheng-Tu 2011, Keselman-Berg 2015, Ruhman-Altman, Jiang-Li-Seidel-DH Lee 2017]
  Basic strategy: spin-charge separation. Spins form SPT, charges are gapless.
- Understanding of exp'l localization in terms of bulk single-particle gap [Grover-Vishwanath 2012, Keselman-Berg 2015].

Example

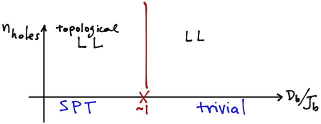
Spin-1 chain
$H = +J_b \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + D_b \sum (S_i^z)^2$ has
Haldane phase.





Dope it! $\mathbf{H} = -t_b \sum_{i\sigma} b^\dagger_{i\sigma} b_{i+i\sigma} + h.c. + J_b \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + D_b \sum (S_i^z)^2$.
Topological Luttinger Liquid should have
spin-half edge states.

[Jiang-Li-Seidel-D-H Lee 2017]



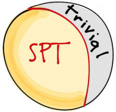
How to tell?

Sharply defined labels for gapless SPTs

Some diagnostics familiar from gapped SPTs work surprisingly well.

- Strange correlator [You-Bi-Rasmussen-Slagle-Xu 2013]:

Make an edge in euclidean spacetime between trivial and nontrivial:
$Z_{\text{strange}} = \langle \Psi_{\text{trivial}} | \Psi_{\text{SPT}} \rangle = \sum_{\sigma,\tau} e^{\mathbf{i}\theta(\sigma,\tau)} = \sum_{\text{dw}} n^{N(\text{dw})} x^{L(\text{dw})}$ is determined by edge CFT (plus bulk data). [Scaffidi-Parker-Vasseur 2017]

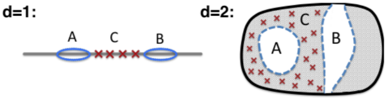


gapped: $x^{-1} = \sqrt{2}, n = 2.$ $c = 1$

gapless: $x^{-1} \to 0, n = 2.$ $c = 2.$

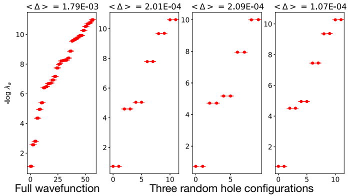
- Nonlocal order parameters [Marvian 2013 (gapped case), Verresen-Thorngren-Jones-Pollmann 2019] :

For unitary, abelian G: in 1d, string operators.
'symmetry flux' $s_{kl} = \text{tr} \rho X_k \prod_{i \in C} U_g(i) Y_l$
acts as a symmetry in region C separating A, B,
X_k acts on A, Y_l acts on B.



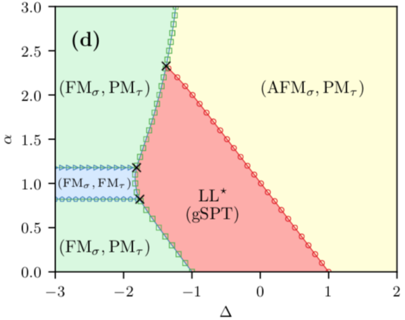
- QDL diagnostic [Ben-Zion, Grover, JM, to appear] :

Measure the gapless component. The remaining wavefunction behaves like an ordinary gapped SPT wavefunction.
In the topological Luttinger liquid phase, measure charge: $\longrightarrow$



Construction by decorating domain walls

[gapped: Chen-Lu-Vishwanath 2013, gapless: Parker-Scaffidi-Vasseur 2017-2018]



Start with a gapless non-topological phase (eg XXZ, with U(1) (to protect gapless bulk) and $\mathbb{Z}_2 \times \mathbb{Z}_2$ (to protect SPTness).)

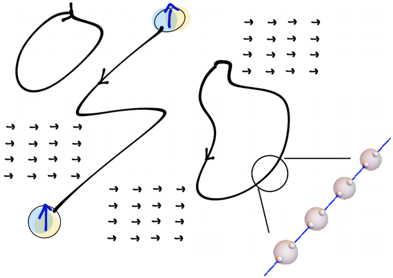
$H_{\text{gSPT}} = U(H_{\text{gTrivial}}(\sigma) - \sum_{i \text{ even}} \tau_i^x)U^{-1}$,
$U \equiv \otimes_{dw(\sigma)_i}(-1)^{(1-\tau_i^z)/2}$.

This produces a stable phase in which an open chain has 2-fold degeneracy (not 4).

Known-to-me 2d examples are gapless by tuning density of decorated domain walls to a critical point. But surely $\exists$ 2+1d fixed points with no symmetric relevant operators for some G.

Definition 2: Gapless states with fractional quasiparticles

- Gapless SETs (symmetry-enriched gapless spin liquids)
- Gapless fracton states



Symmetry-enriched 3d U(1) spin liquids

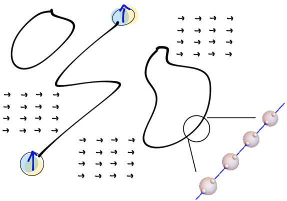
[Wang-Senthil, 2013, 2016, Zou-Wang-Senthil 2018]

Gapless emergent photon, symmetry-protected edge states (sometimes).

So far: assuming gapped E and M excitations, classifications for various G.

Strategy: starting with a $\mathsf{U}(1)$ spin liquid with gapped charges, put one of the charges in an SPT phase. EM duality $\implies$ not all different.

Or: starting with a $\mathsf{U}(1) \times \mathsf{G}$ SPT, gauge the $\mathsf{U}(1)$.



[like Ben-Zion,Das,JM 2015]

Here, too, solvable models can be made by decorating electric flux lines.

$H = U H_0 U^{-1}$ where

$H_0 = -\sum_j X_j - \sum_j (\Delta n)_j^2 - \sum_\square \cos(\Delta \times a)_\square$

$\mathsf{U}(1)$ string nets plus decoupled qbits

$U = \otimes_\ell C_{n_\ell} U_\ell \equiv \sum_{\{n_\ell\}} |\{n_\ell\}\rangle\langle\{n_\ell\}| \otimes U_\ell^{n_\ell}$,

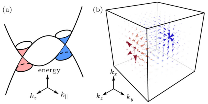
$U_{\langle ij \rangle} \equiv \mathbf{CZ}_{ij}$ Electric charges are Kramers' doublets ($E_{bT} M_b$ phase).

Lots more to do here: realization in pyrochlore spin ice ($Yb_2Ti_2O_7$)?, lattice symmetries, other gauge groups, gapless charged matter, decorate string defects? Can we decorate a Fermi liquid?

Definition 3: Gapless states with (bulk) 't Hooft anomalies

Here the idea is that topology protects the gaplessness from small perturbations.

- edge theories of SPTs [Max's talk]
- many familiar CFTs
- topological semimetals and superconductors
- some much more prosaic states.

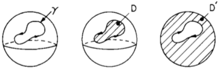


[Image: Lu Hai-Zhou, Shen Shun-Qing]

Anomalies in 2d CFT

Many AF spin chains $H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + \cdots$ can be described using non-Abelian bosonization $S = -\frac{1}{\lambda} \int_\Sigma d^2x \mathrm{tr} g^{-1} \partial_\mu g g^{-1} \partial^\mu g + k\Gamma_{\rm wz} + \cdots$.

$k \in \mathbb{Z}$ in order that $\Gamma_{\rm wz} = \frac{1}{24\pi^2} \int_{B, \partial B = \Sigma} \epsilon^{\mu\nu\rho} \mathrm{tr} g^{-1} \partial_\mu g g^{-1} \partial_\nu g g^{-1} \partial_\rho g$ is well-defined. $k = 2S$.



[Furuya-Oshikawa 2015]: a mixed anomaly between $\mathsf{SU}(2)$, translations and $\mathbb{Z}_2 : g \to -g$ forbids gauging the $\mathbb{Z}_2$ for odd-half-integer spin chains.

Anomaly diagnosed by failure of modular invariance of the resulting orbifold CFT.

This anomaly is conserved under RG, and prevents changing the parity of the level k (measurable by Raman scattering).

$\exists$ many examples in the string theory literature of such forbidden orbifolds.

Failure of mutual locality of the operator algebra of the would-be orbifold CFT is a more stringent criterion.

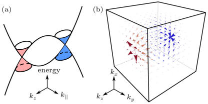
General coherent story for rational CFT [Meng Cheng, Williamson, to appear]: a formula for the anomaly in terms of CFT data.

Weyl semimetal [reviews: Turner-Vishwanath 2013, Armitage-Mele-Vishwanath 2018]

Weyl nodes can only⋆ disappear by annihilating each other [Horava].

Consequence: surface Fermi arcs.

Edge states at $k \neq k_{\text{FS}}$ can't mix.



[Image: Lu Hai-Zhou, Shen Shun-Qing]

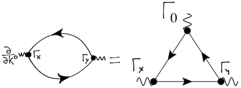
Definition away from free fermions: [Witczak-Krempa-Knap-Abanin 2014] in terms of Berry flux of $H_t(\vec{k}) \equiv G(\omega = 0, \vec{k})^{-1} \equiv H_0(\vec{k}) + \Sigma(\omega = 0, \vec{k})$.

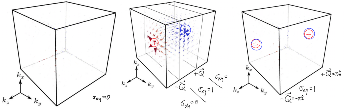
Bulk (chiral, ABJ) anomaly manifests in transport:

$\sigma_{xy} \overset{\text{Kubo}}{=} \partial_\omega \Pi_{xy}|_{\omega\to 0} = \frac{e^2}{2\pi h} \int đk_z C_z(k_z)$

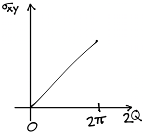
Using Ward identity: $\Gamma_\mu = \partial_\mu G^{-1}$

$C_z(k_z) = \frac{\pi}{3} \int đ^3 k_{x,y,0} \epsilon_{\mu\nu\rho z} \text{tr} G \partial_\mu G^{-1} G \partial_\nu G^{-1} G \partial_\rho G^{-1}$



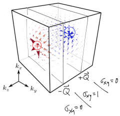


$\implies$ Weyl SM is an intermediate between trivial insulator and layered Chern insulator.



Weyl semimetal

More on chiral anomaly: Translations in the direction separating the Weyl nodes act by chiral rotation:
$\psi_{L/R} \to e^{\pm \mathbf{i} Q} \psi_{L/R}$

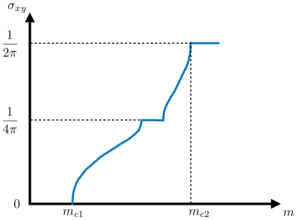


[Wang-Gioia-Burkov 2019] $z \equiv \mathbb{Z}$ gauge field for translations [Else-Thorngren],
$S[z, A] \ni 2\pi\nu \int_{5d} z \cup F \wedge F$

A new ingredient: ν is not quantized! A state can be 'a little bit anomalous'. Not protected if $v_F Q \sim \Delta H$.

* There is a gapped (FQHE) state which saturates this anomaly.

Break $\mathsf{U}(1)$ by superconducting pairing, try to restore $\mathsf{U}(1)$ by proliferating vortices, only charge-4 vortex line is featureless.



(m = magnetization, determines $\vec{Q}$)

[Wang-Gioia-Burkov]

LSM anomaly

Any time a gapless state is the resolution of an LSM alternative.

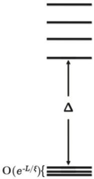
For example: electrons at fractional filling ν with lattice translation symmetry and charge conservation.

[Senthil, Song-Vishwanath-Wang-He] $S[z, A] \ni 2\pi\nu \int_{5d} z^{(x)} \cup z^{(y)} \cdots \wedge F$

Could be a Fermi liquid.

Definition 4: Gapless topological order

Topological order ≡ a collection of (approximate) groundstates which can't be distinguished by local operators:
$\langle a|\mathcal{O}_{\text{local}}(x)|b\rangle \sim e^{-L^{\alpha>0}}$

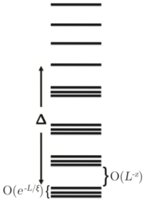


[Image: Bonderson-Nayak]

Topological splitting $e^{-L^{\alpha>0}} \ll \frac{1}{L^{z>0}}$ gapless splitting

Gapless topological order

[Senthil-Fisher, 2000, Barkeshli-JM, 2012 (unpublished), Bonderson-Nayak 2013]



Find the spectrum on T^d.

A problem in gauge theory in finite volume. [XGW-Niu 1993, Shenker et al 2012]

Examples:

- Laughlin FQH state coupled to phonons. ✓
- Laughlin FQH state coupled to 3d photon has $\Delta \sim \alpha L^{-1}$ [Bonderson-Nayak].
- HLR state. $\Psi_e = fb$, b in $\nu = 1/q$ Laughlin state.

$$L_{\text{eff}}(a, \tilde{a}, f) = -\frac{q}{4\pi}\tilde{a}\partial\tilde{a} + \frac{1}{2\pi}a\partial\tilde{a} + L_m(f, a)$$

Zero modes: $X \equiv \oint_{C_x} a, Y = \oint_{C_y} a$

$$S = -2\pi q \tilde{X}_i \partial_t \tilde{X}_j \epsilon_{ij} + 2\pi X_i \partial_t \tilde{X}_j \epsilon_{ij} + \underbrace{S[\chi_0, X]}_{\text{lifts } X} \to -2\pi q \tilde{X}_i \partial_t \tilde{X}_j \epsilon_{ij}.$$

$\rightsquigarrow$ q-fold exp'l degeneracy.

- HLR* state: put b in a *nonabelian* incompressible $\nu = 1/2$ state (eg the orbifold state [Barkeshli-XGW 2010]).

Proposal [MB]: lift all degeneracies associated with the fusion channels of gapless anyons. Anyons with trivial braiding with those will still produce exponential splitting.

Honorable mention

- [Simon-Rezayi-Cooper 2007] Haffnian, Gaffnian... FQH wavefunctions based on non-unitary or irrational CFT
- [Fisher-Levin 2009] 3d FQH state
- [Freedman-Hastings-Nayak-Qi 2011] "Weakly coupled non-Abelian anyons in three dimensions"
- [Fitzpatrick-Kachru-Kaplan-Katz-Wacker 2012] $z = 2$ theory of anyons

The end.

Conclusion: lots to do!

Thanks for listening.