Higher symmetries, p-string condensation and fractons

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Why are fracton phases interesting?

- Quantum information properties/applications (*e.g.* finite-temperature quantum memory)
- New class of quantum phases of matter / new phenomena
- **Relation with quantum field theory (QFT)**
- Realization: new kinds of local constraints

Big question: what is the relationship between phases of quantum matter and QFTs?

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Fracton phases are a challenge to our understanding of this question, *e.g*....

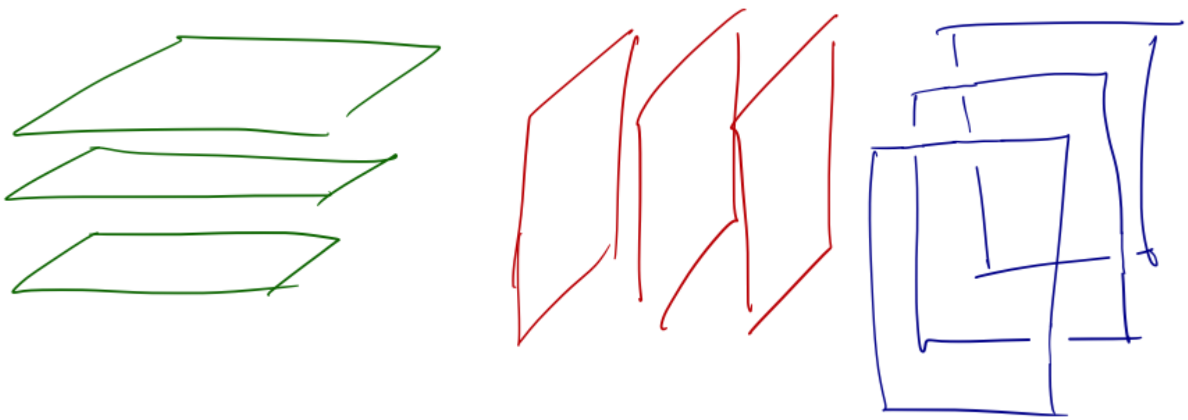
- (Naive) expectation:
  gapped phase of matter → low-energy TQFT description

  But gapped *fracton* phases lack an (ordinary) TQFT description

- Naive RG on a typical fracton phase → exponentially diverging number of superselection sectors / volume
  … infinite number of fields??

“Rigid” geometrical structure

- We are used to putting QFTs on a variety of spaces/spacetimes with some structure, *e.g.* Riemannian metric, orientation, spin structure, …
- Fracton phases seem to require geometrical structures that are more “rigid” than what we are used to in this context
- Example: X-cube model (and related phases) and foliation structure (Shirley, Slagle, Z. Wang, X. Chen)



- Open question: what geometrical structure underlies the various U(1) tensor gauge theories?

Coupling & condensation/gauging constructions

- Coupling & condensation: start with a collection of ordinary (non-fracton) systems, couple them in some "unusual" way and condense some objects to get fractons.
  Ma, Lake, Chen, MH; Vijay; Radzihovsky & MH; …

- Gauging constructions: start with an ordinary gauge theory or a trivial gapped system with some "unusual" symmetry, gauge it to get fractons.
  Vijay, Haah, Fu; Williamson; Devakul, You, Burnell, Sondhi; Shirley, Slagle, Chen; Williamson, Bi, Cheng; …

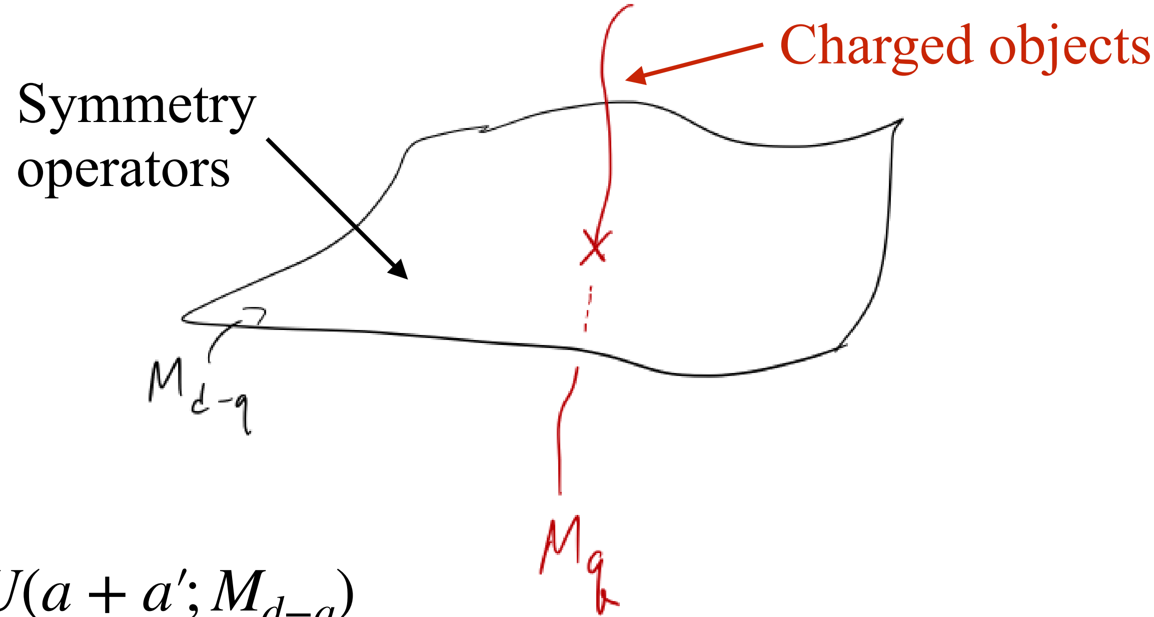
- Useful for understanding QFT ↔ fractons relationship?

- Needed: more precise understanding of what is "unusual" in these constructions

Higher-form symmetries

- q-form A-symmetry in d-dimensional space:

Abelian group



$$U(a;M_{d-q})U(a';M_{d-q}) = U(a+a';M_{d-q})$$

$$U(a;M_{d-q})U(a';M'_{d-q}) = U(a';M'_{d-q})U(a;M_{d-q})$$

- Condense charged q-dimensional objects: spontaneously break symmetry
  - A finite, q=1: deconfined phase of A-gauge theory
  - A=U(1), q=1: Coulomb phase of U(1) gauge theory

Faithful / non-faithful q-form symmetries

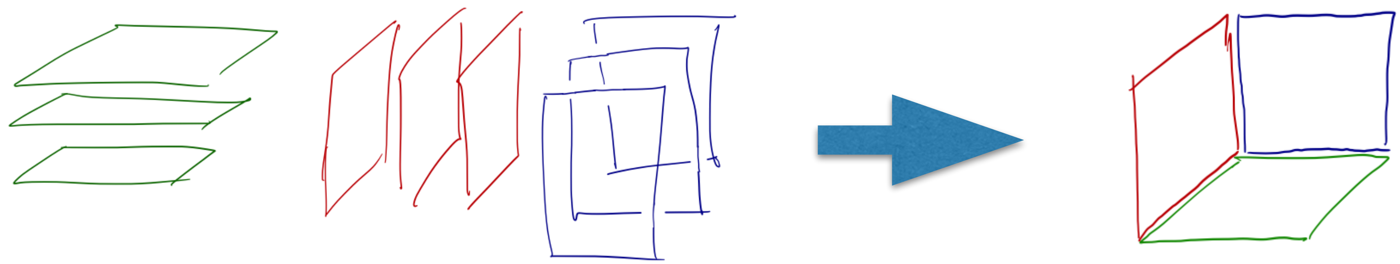
- This talk: focus on UV q-form symmetries, hold at all scales, symmetry operators *not* topological
- "Faithful" q-form symmetry: different submanifolds always have different symmetry operators

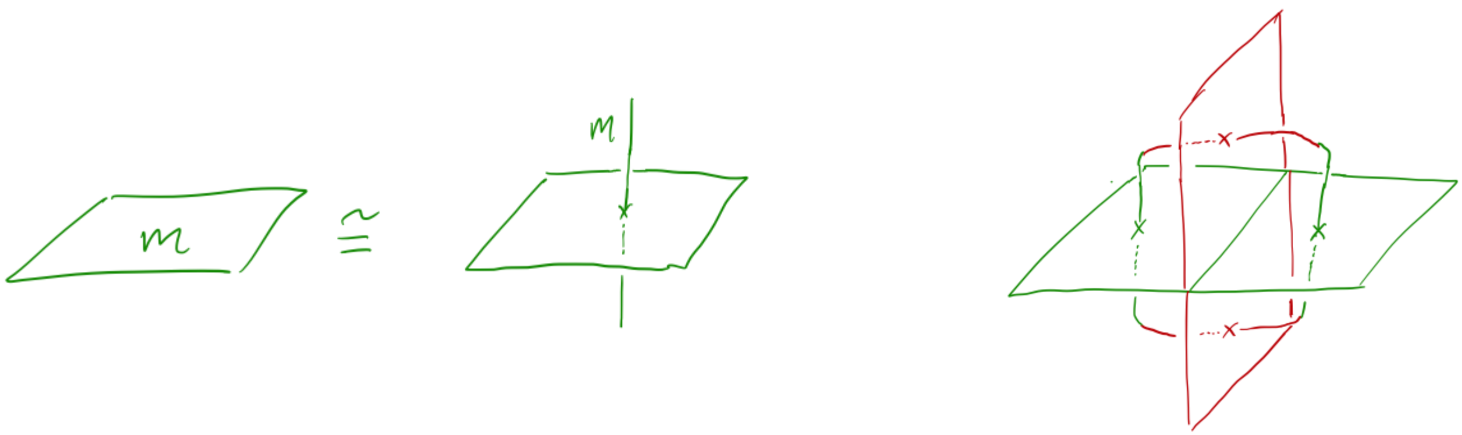
$$M_{d-q} \neq M'_{d-q} \implies U(a; M_{d-q}) \neq U(a; M'_{d-q}) \qquad (a \neq 0)$$

- In particular, $U(a; M_{d-q}) \neq 1$ if $a \neq 0$ and $M_{d\text{-}q}$ is non-empty

p-string condensation and 1-form symmetry

- Choose three foliations of the spatial 3-torus with d=2 leaves
- Discretize the foliation structure (get a cell structure) & put down a d=2 toric code (Z_2 gauge theory) on each leaf





A closed m-string

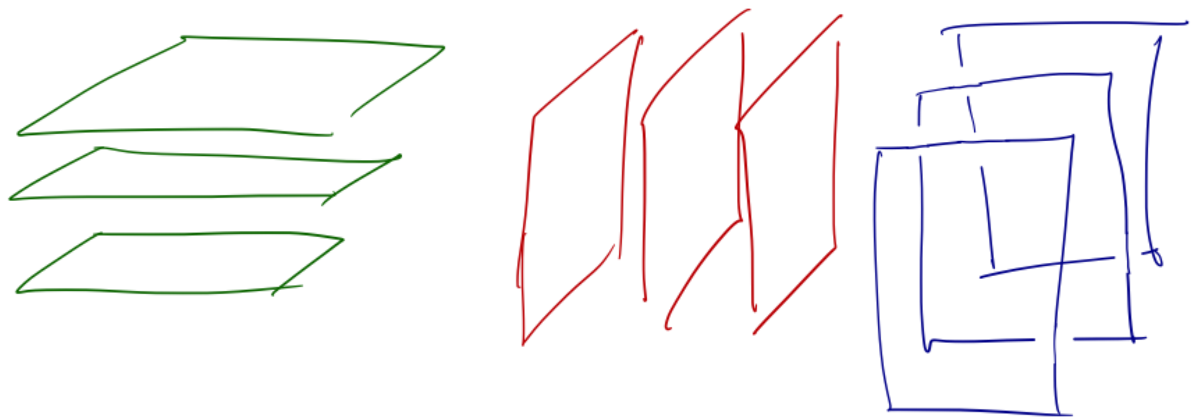
- m-particles (gauge fluxes) live on 2-cells, or dual 1-cells
- Natural to impose a Z_2 1-form symmetry, where strings of m-particles are the charged objects

p-string condensation and 1-form symmetry

- Condense closed *m*-strings → X-cube fracton phase
- But something "unusual" has happened, how to characterize it?
- The 1-form symmetry is non-faithful:
  symmetry operators are trivial when M_2 is a leaf of the foliation
- Formally, this symmetry can be obtained as a quotient of a faithful Z_2 1-form symmetry, by the subgroup generated by taking M_2 all leaves. Specifying this subgroup requires the "rigid" geometrical structure of a foliation.

Complementary gauging construction

- Start with a trivially gapped *d=3* system with a (faithful) Z_2 1-form symmetry, and choose a (discretized) foliation structure
- Symmetry operator on leaf M_2 generates Z_2 subgroup of the symmetry. View this as a 0-form Z_2 symmetry on the leaf



- Gauge all these Z_2 subgroups, get layers of *d=2* Z_2 gauge theories on the leaves
- There is a residual non-faithful 1-form symmetry given as a quotient by the gauged subgroup. This is the *same* symmetry as in the coupling construction.

Complementary gauging construction

- Under the (non-faithful) 1-form symmetry, electric charges of the *d*=2 gauge theories organize into the 1-dimensional charged objects (electric-magnetic dual of coupled-layer construction)
- Condense the 1d charged objects, get the X-cube fracton phase

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- Alternatively, can start with a *d=3* Z_2 toric code, consider its electric 1-form symmetry, and follow the same gauging procedure (Williamson, Bi & Cheng arXiv:1809.10275).
- This gets one directly to the X-cube fracton phase
- Here one *first* breaks the 1-form symmetry (in the *d=3* toric code), and then gauges a certain subgroup — steps reversed from above

Generalizing p-string condensation

1. Start with some collection of "ordinary" systems, *e.g.* gauge theories, S_α
2. Impose a 1-form symmetry where some point-like charges of the S_α's organize into the 1d charged objects
3. Condense the charged objects

- "Rigid" geometrical structure may be needed at step 1, step 2, or at both steps
- Various generalizations can be explored: consider q-form symmetry with $q > 1$, multiple symmetries, non-point-like excitations of the S_α's, …

Generalizing gauging constructions

1. Start with a trivially gapped system with a faithful 1-form symmetry
2. Using some rigid geometrical structure, identify and gauge some subgroup of this symmetry
3. Condense the 1d objects charged under the residual 1-form symmetry

- Steps 2 and 3 can be swapped
- Various generalizations possible, as for coupling constructions

p-string/gauging constructions of U(1) tensor gauge theories

- Recall the rank-2 scalar charge theory in d dimensions:

  Electric field E_{ij} Gauss law $\partial_i \partial_j E_{ij} = \rho$

  Charge and dipole moment are conserved
  $\rightarrow$ electric charges are fractons

- p-string and gauging constructions for this theory have been discussed recently (from different points of view)

  Gauging: D. Williamson, Z. Bi, M. Cheng. arXiv:1809.10275

  p-string: Radzihovsky & MH. arXiv:1905.06951

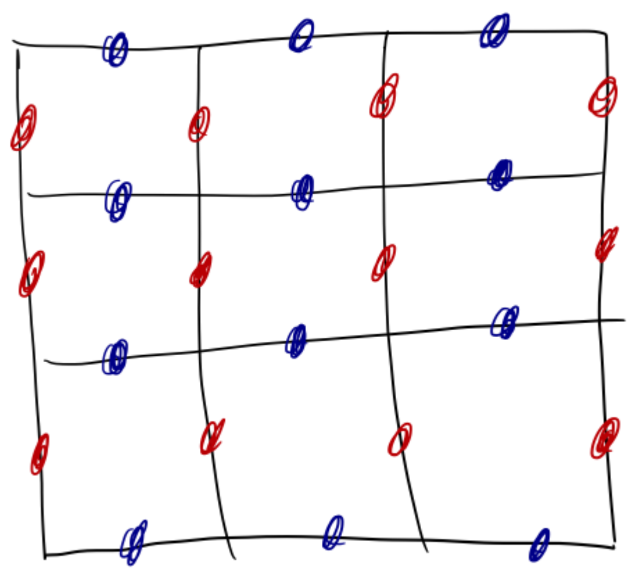
- I'll give an account of these constructions from the higher-symmetry point of view
  - Apparent rich possibilities for other similar constructions
  - Foliation structure underlying the rank-2 scalar charge theory

Coupling construction of rank-2 scalar charge theory

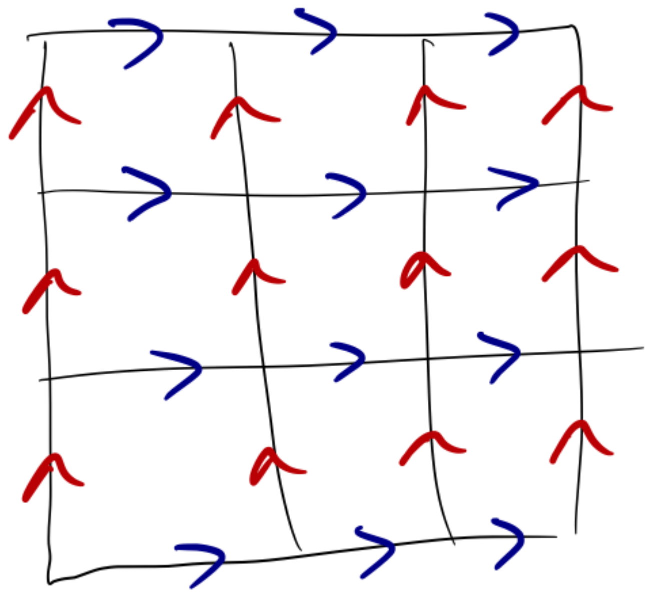
- Focus on d=2 for concreteness, spatial 2-manifold M
- Introduce two U(1) gauge fields E_{red} and E_{blue}

$$\nabla \cdot E_{red} = \rho_{red} \qquad \nabla \cdot E_{blue} = \rho_{blue}$$

- Want to view ϱ_{red}, ϱ_{blue} charge configurations as configurations of oriented strings
- Introduce a cell structure for M, each 1-cell is labeled red or blue. Red/blue charges live on the red/blue 1-cells.



Coupling construction of rank-2 scalar charge theory



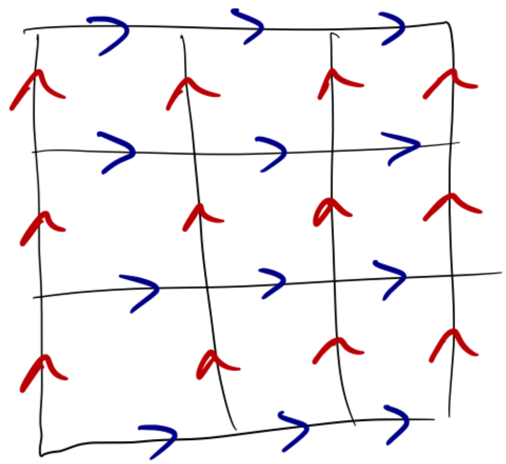
- Introduce orientations for each 1-cell:
  charge configurations ≅ oriented string configurations

- Impose a U(1) 1-form symmetry

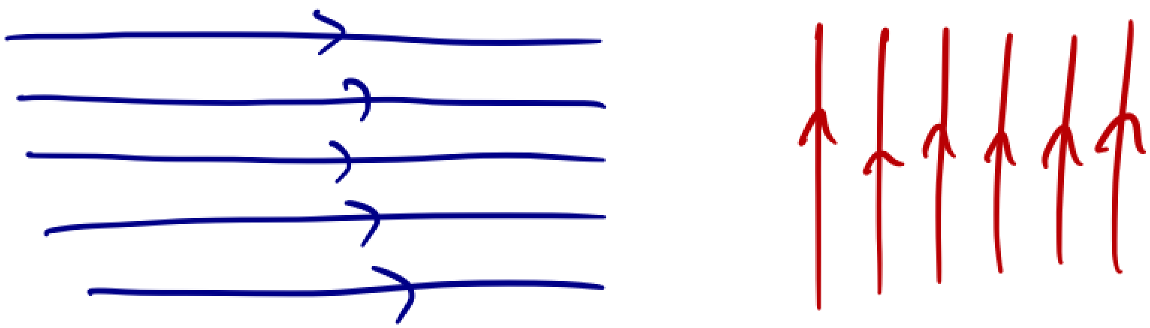
- Choose orientations so that contractible loops have $Q_{red} = Q_{blue} = 0$.
  $\Rightarrow$ Small loops locally createable $\Rightarrow$ Loop condensation possible

- Condense the loops & spontaneously break the 1-form symmetry

Scalar charge theory and foliation structure



- The cell structure we've already illustrated gives the scalar charge theory at low-energy (caveat: in d=2 need to be careful about symmetries)



- This cell structure naturally comes from two foliations of M by oriented 1-dimensional manifolds

Gauging construction

- The coupling construction we've just sketched can be viewed as a gauging construction

- Discussion can be formalized using cellular homology/ cohomology:

  → cellular chains ≅ red/blue charge configurations
  → cellular 1-cocycles ≅ 1-form symmetry operators

- Q_{red} and Q_{blue} are 1-cocycles, e.g. $Q_{red}(\partial c_2) = \delta Q_{red}(c_2) = 0$

- Q_{red} and Q_{blue} are thus subgroups of the 1-form global symmetry

- So instead we could have started with a system with U(1) 1-form global symmetry, and gauged the Q_{red} and Q_{blue} subgrops

Generalizations so far

- Scalar charge theory in $d > 2$ obtained by straightforward generalization: need d colors of U(1) gauge fields, still a single U(1) 1-form symmetry

- Geometrical structure: d foliations of d-dimensional space by oriented 1-manifolds

- "Vector charge theory": $E_{ij} = E_{ji} \qquad \partial_i E_{ij} = \rho_j$

- Can be obtained using d U(1) 1-form symmetries and $\binom{d}{2}$ colors of U(1) gauge fields

Summary

- Higher-form symmetry $\rightarrow$ generalized p-string condensation / gauging constructions of fracton phases

- Large variety of constructions are possible (we are beginning to explore some of them)

- Geometrical structure underlying the rank-2 scalar charge theory: d foliations of oriented 1-manifolds. Consequences?

- Outlook: we can learn a lot about fracton phases by thinking carefully about their (higher) symmetries.
(See also Nati Seiberg's talk Friday morning.)