Dynamics of composite fermions

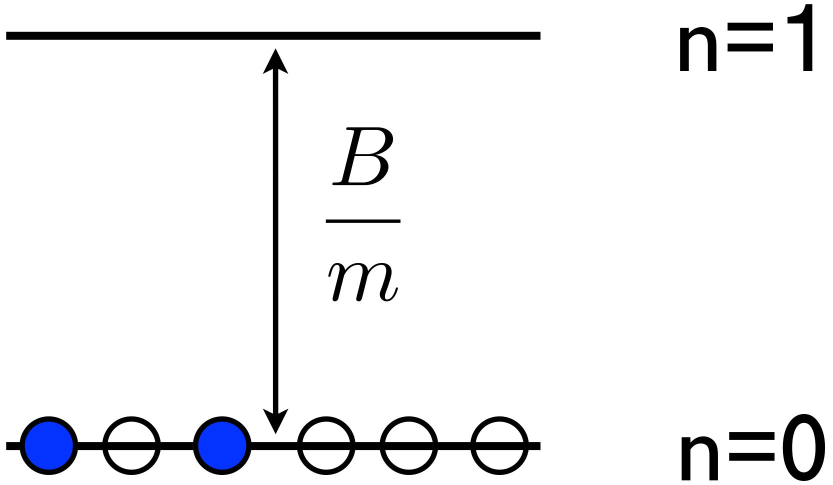
Dam Thanh Son (University of Chicago)
UQM Kickoff meeting, 09/12/2019

Plan

- Fractional quantum Hall effect
- Dirac composite fermion
- Hydrodynamics with “dynamical metric”

Lowest Landau level limit

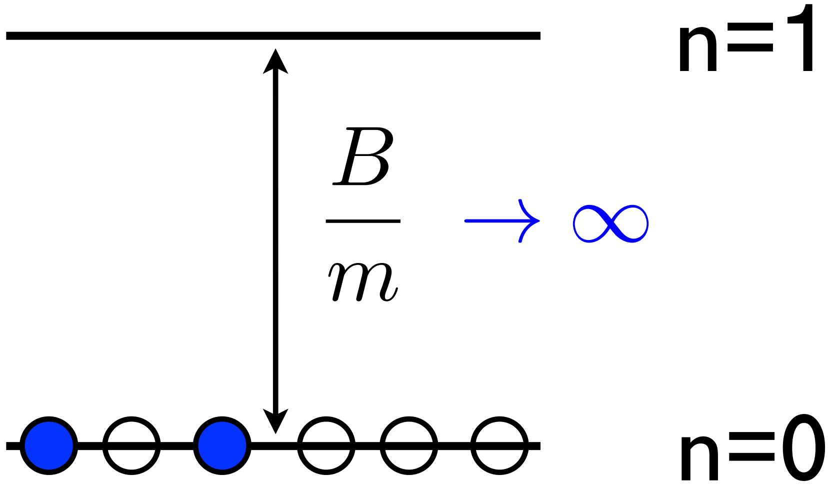
$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$



Lowest Landau level limit

$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

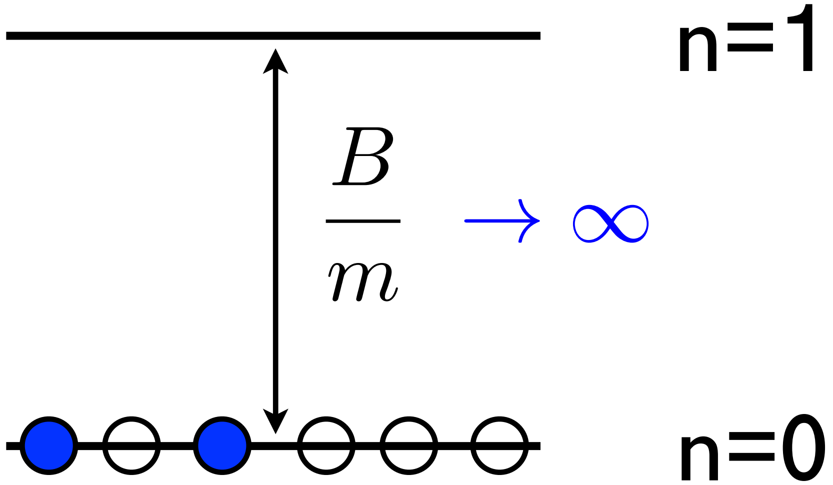
$m \to 0$



Lowest Landau level limit

$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

$m \to 0$



$$H = P_{\mathrm{LLL}} \sum_{a,b} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

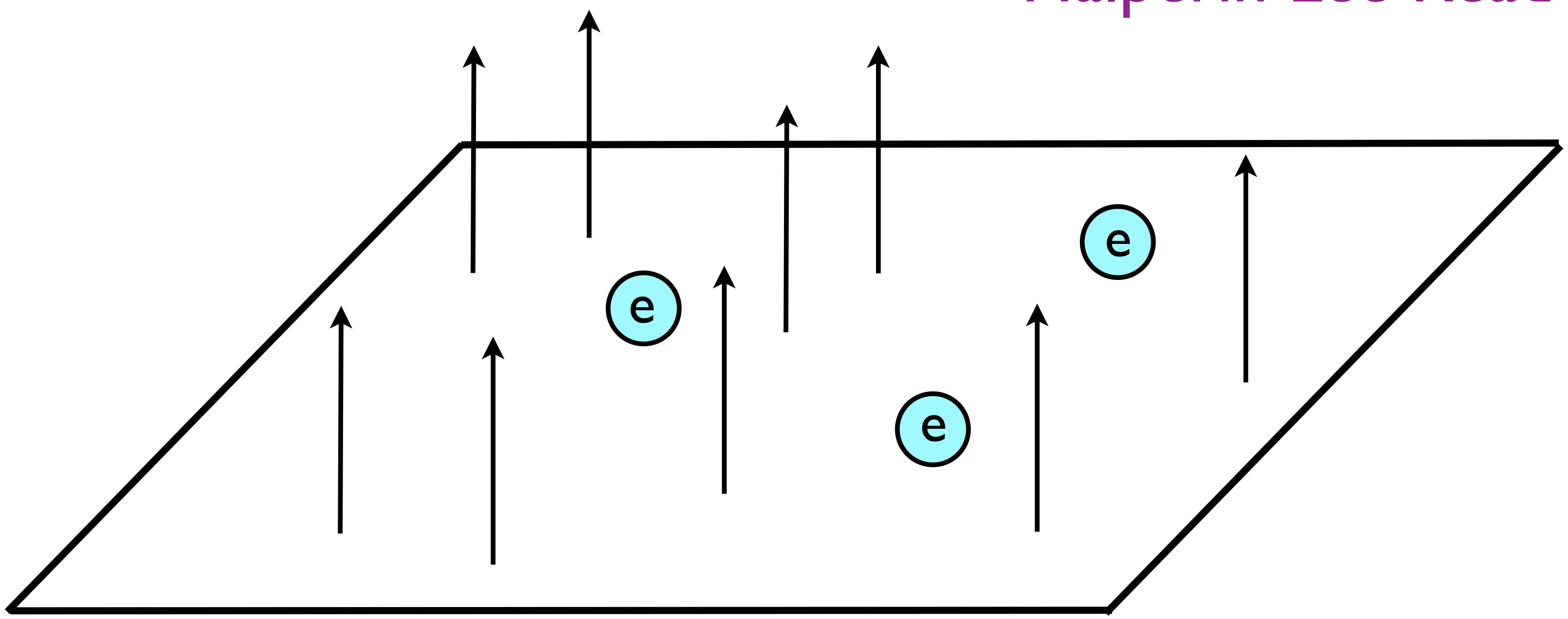
Projection to
lowest Landau level

Composite fermion

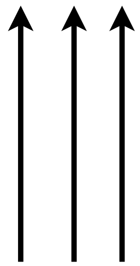
- A new quasiparticle: “composite fermion” = electron + 2 flux quanta

Flux attachment

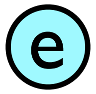
Jain, Lopez Fradkin, Ovchinnikov,
Halperin Lee Read ~ 1990



$\nu = \frac{1}{3}$

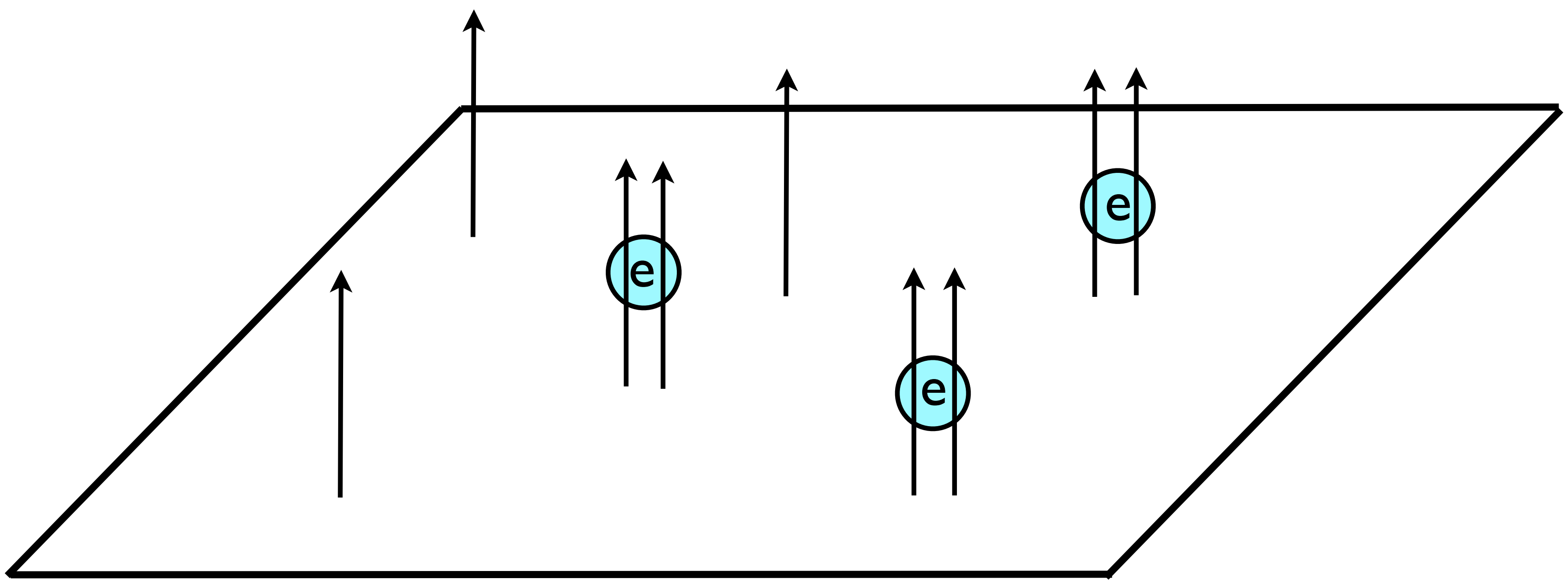


per

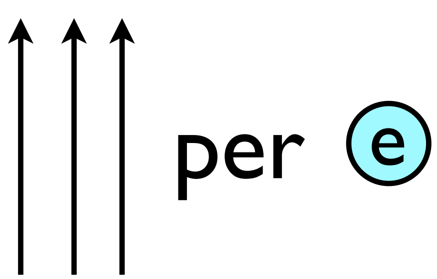


Flux attachment

Jain, Lopez Fradkin, Ovchinnikov,
Halperin Lee Read ~ 1990

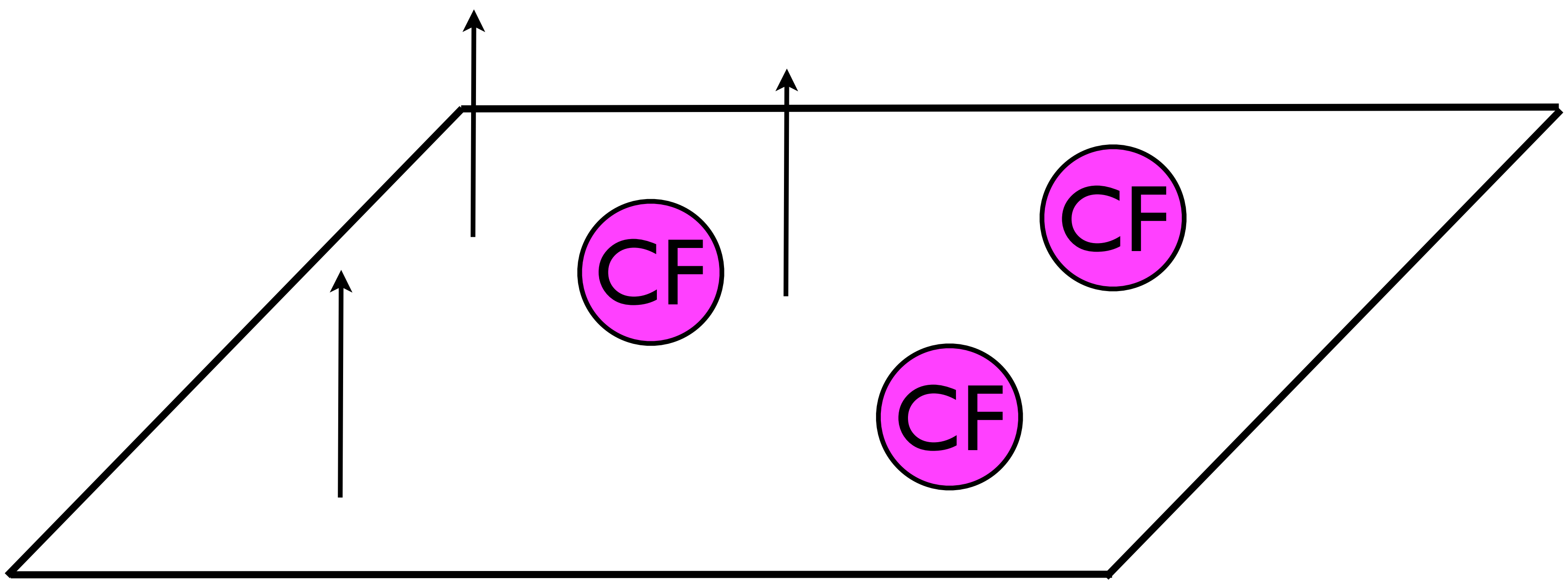


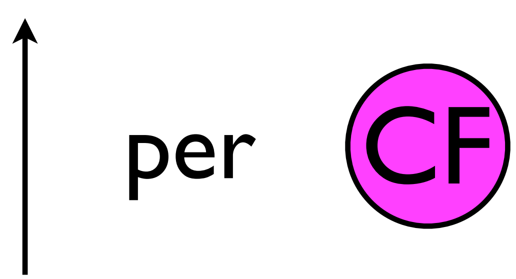
$$\nu = \frac{1}{3}$$

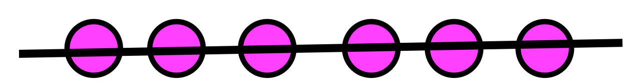


Flux attachment

Jain, Lopez Fradkin, Ovchinnikov,
Halperin Lee Read ~ 1990







full LL: gapped state

(CF = composite fermion)

HLR field theory

$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{2}\frac{1}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

$$b = \nabla \times a = 2 \times 2\pi\psi^\dagger\psi$$ "flux attachment"

mean field: $$B_{\text{eff}} = B - b = B - 4\pi n$$

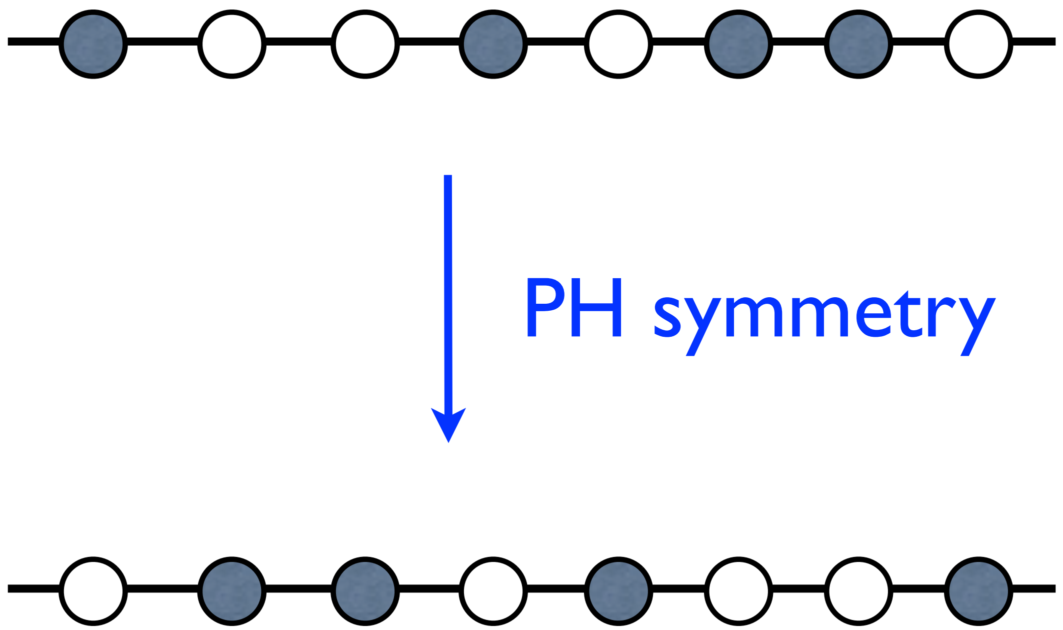
$$\nu = \frac{1}{2} \qquad B_{\text{eff}} = 0$$

low-energy dof: ψ excitations near Fermi surface

Problems of HLR theory

- The problem of energy scale (CF effective mass remains finite when me->0)
- The lack of particle-hole symmetry
  - solved by Dirac CF theory

Particle-hole symmetry



$$\Theta|\text{empty}\rangle = |\text{full}\rangle$$

$$\Theta c_k^\dagger \Theta^{-1} = c_k$$

$$\Theta\, i\, \Theta^{-1} = -i$$

$$\nu \to 1 - \nu$$

exact symmetry the Hamiltonian on the LLL, when mixing of higher LLs negligible

$\nu = 1/2$ maps to itself

Fermionic particle-vortex duality

DTS; Metlitski, Vishwanath; Wang, Senthil 2015

Conjecture: free fermion = "QED" in 2+1 D

physical EM field

Theory 1: $$\mathcal{L} = i\bar{\psi}_e \gamma^\mu (\partial_\mu - iA_\mu)\psi_e$$

Theory 2: $$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu \partial_\nu a_\lambda$$

emergent U(1) gauge field

Theory 1 ψ_e	Theory 2 ψ
magnetic field	density
density	magnetic field

Particle-vortex duality

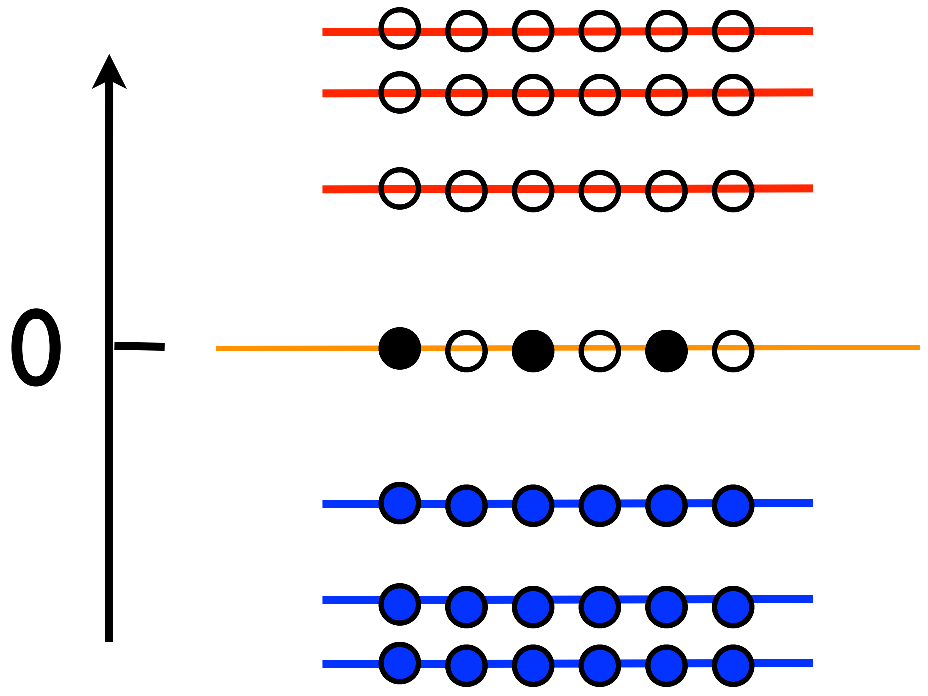
$$S = \int d^3x \, i\bar{\psi}_e \gamma^\mu (\partial_\mu - iA_\mu)\psi_e$$

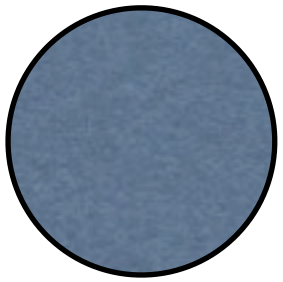
$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda \right]$$

$$\rho = \frac{\delta S}{\delta A_0} = -\frac{b}{4\pi} \qquad\qquad \frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \bar{\psi}\gamma^0\psi \rangle = \frac{B}{4\pi}$$

$$\|$$

$$\bar{\psi}_e \gamma^0 \psi_e = \psi_e^\dagger \psi_e$$





Theory 1 in magnetic field
zero charge density

Theory 2 at finite density
and zero magnetic field

Half-filled Landau level

Fermi liquid

ψ_e = electrons

ψ = "composite fermion"

LLL projection

- The energy scale problem left unsolved by Dirac CF theory
  - origin of the Fermi velocity
- Lack of LLL projection: is it a problem with low-energy effective theory?
  - not necessarily, unless the LLL has consequences for low-energy physics
  - one of such consequences: density-density correlator ~ q^4, not q^2

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{j} = 0 \qquad \rho \sim \frac{qj}{\omega} \sim \frac{q^2 T}{\omega B}$$

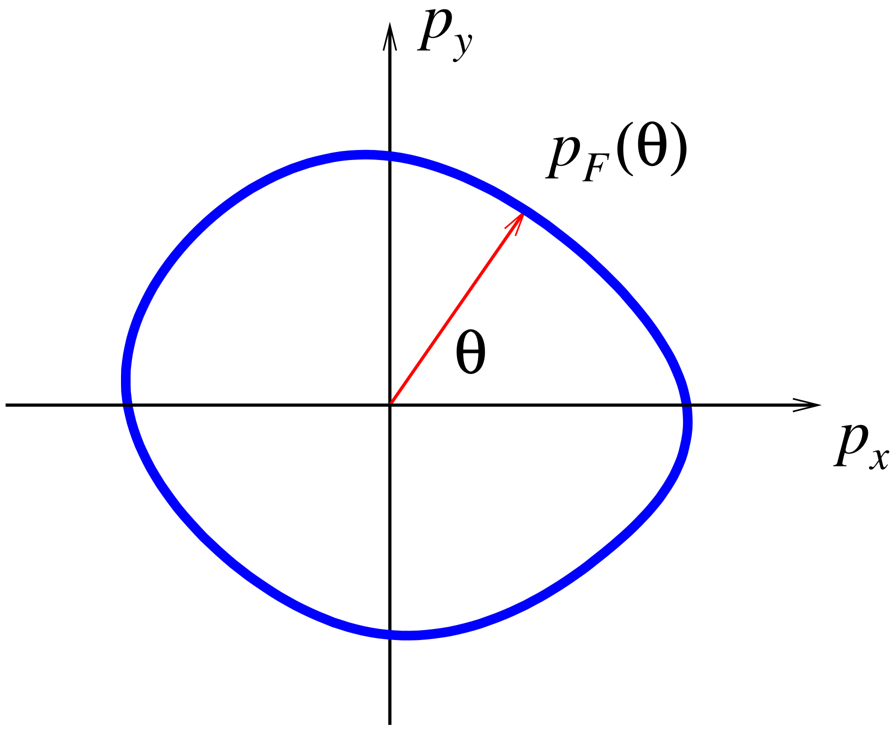
$$\frac{\partial}{\partial t}(m\mathbf{j}) + \boldsymbol{\nabla} \cdot \mathbf{T} = \mathbf{j} \times \mathbf{B} \qquad j \sim \frac{qT}{B}$$

- Needed: a composite fermion theory that satisfies all constraints placed by LLL projection
- One requirement: CF has electric dipole moment with respect to the physical EM field
- It is not clear what is the place of this requirement in the field-theory duality

A hydrodynamic theory of the composite fermions

- Ref: arXiv:1907.07187
  - related work: A.Gromov & DTS "bimetric theory", Haldane's dynamical gravity

Bosonic excitations



Low-energy, long-wavelength excitations: fluctuations of the shape of the Fermi surface

$$p_F(t,\mathbf{x},\theta) = p_F^0 + \sum_{n=-\infty}^{\infty} u_n(t,\mathbf{x})\, e^{-in\theta}.$$

One scalar field per spin

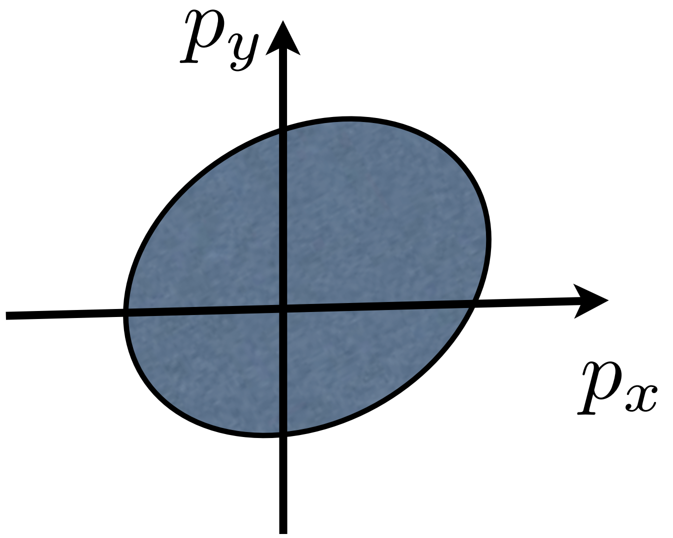
At low momenta we can limit ourselves to a few lowest modes

$$q\ell_B \ll 1/N \qquad \nu = \frac{N}{2N+1}$$

(length >> CF semiclassical orbit)

Nematic hydrodynamics

- Degrees of freedom:



  - density

$$n(\mathbf{x}) = \int \frac{d\mathbf{p}}{(2\pi)^2} f(\mathbf{x}, \mathbf{p})$$

  - momentum density

$$\pi_i(\mathbf{x}) = \int \frac{d\mathbf{p}}{(2\pi)^2} p_i f(\mathbf{x}, \mathbf{p})$$

  - effective metric

$$\int \frac{d\mathbf{p}}{(2\pi)^2} p_i p_j f(\mathbf{x}, \mathbf{p}) = \frac{\pi_i \pi_j}{n} + \pi n(\mathbf{x}) G_{ij}(\mathbf{x})$$

$$\sqrt{\det G} = n$$

Hydrodynamics

Landau 1941

- Hydrodynamics can be formulated as a dynamical system with the Poisson brackets

$$\{\pi_i(\mathbf{x}),\, n(\mathbf{y})\} = n(\mathbf{x})\partial_i\delta(\mathbf{x}-\mathbf{y})$$

$$\{\pi_i(\mathbf{x}),\, \pi_j(\mathbf{y})\} = [\pi_j(\mathbf{x})\partial_i + \pi_i(\mathbf{y})\partial_j]\delta(\mathbf{x}-\mathbf{y})$$

- and Hamiltonian

$$H = \int d\mathbf{x}\left[\frac{1}{2m}\frac{\vec{\pi}^2(\mathbf{x})}{n(\mathbf{x})} + \epsilon(n(\mathbf{x}))\right]$$

$$\dot{n} = \{H,\, n\}$$

$$\dot{\pi}_i = \{H,\, \pi_i\}$$

Chiral metric hydro

$$\{G_{ij}(\mathbf{x}),\, \pi_k(\mathbf{y})\} = (G_{ik}(\mathbf{x})\partial_j + G_{jk}(\mathbf{x})\partial_i + \partial_k G_{ij})\delta(\mathbf{x}-\mathbf{y})$$

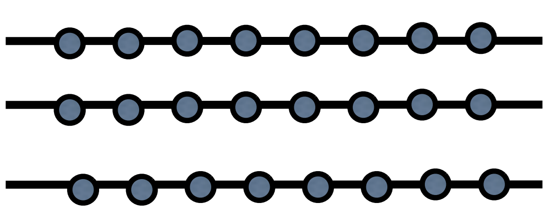
$$\{G_{ij}(\mathbf{x}),\, G_{kl}(\mathbf{y})\} = -\frac{1}{s}(\varepsilon_{ik}G_{jk} + \varepsilon_{il}G_{jk} + \varepsilon_{jk}G_{il} + \varepsilon_{jl}G_{ik})\delta(\mathbf{x}-\mathbf{y})$$

- $(\det G)^{1/2} = n$

s related from the Hall viscosity

= average "orbital spin" of composite fermion

$$s = \frac{1}{N+\frac{1}{2}}\left(\frac{1}{2}\cdot 0 + 1 + 2 + \cdots + N\right) = \frac{N(N+1)}{2N+1}\,.$$



$n = 1$

Hydrodynamic equation

$$H = H_0[n, \pi_i, G_{ij}] + \int d\mathbf{x} \left(-a_0 n + \frac{\varepsilon^{ij} E_j}{B} \pi_i \right)$$

CF dipole moment

$$\dot{A} = \{H,\, A\}$$

Electron density

$$\rho = \frac{B-b}{4\pi} - \epsilon^{ij}\partial_i \left(\frac{\pi_j}{B}\right)$$

dipole contribution

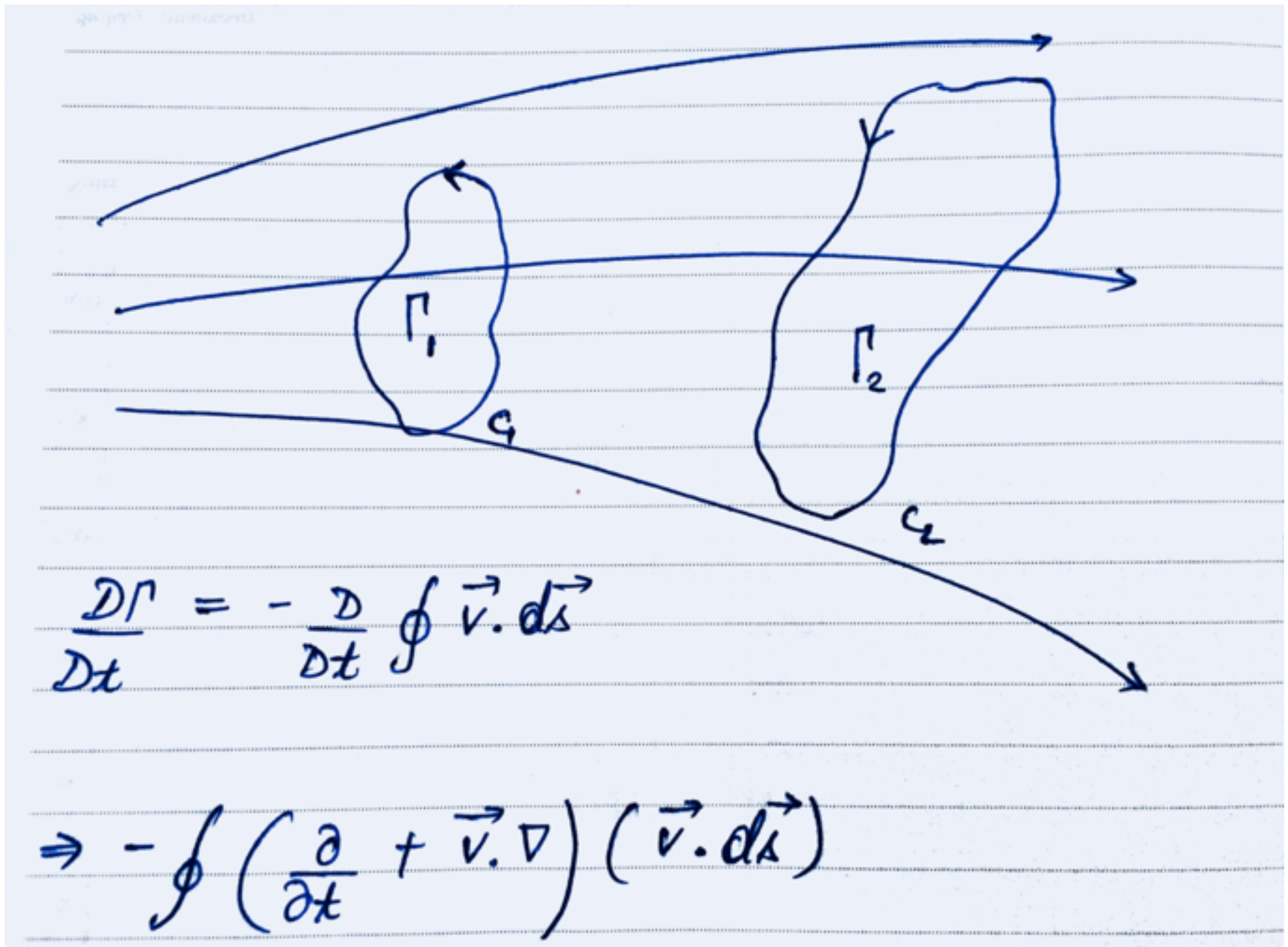
$$= n - \frac{b+\omega}{4\pi}$$

$$\omega = \vec{\nabla} \times \left(\frac{\vec{\pi}}{n}\right)$$

vorticity

Kelvin's circulation theorem

1869



engineering.stackexchange.com

- In ideal hydrodynamics vorticity is carried with the flow

$$\dot{\omega} + \vec{\nabla} \cdot (\omega \vec{v}) = 0$$

vorticity

$$\omega = \vec{\nabla} \times \left(\frac{\vec{\pi}}{n} \right)$$

- Leads to an infinite number of conserved quantities (Casimirs of the Poisson algebra)

Kelvin's circulation theorem

- In the presence of magnetic field and metric degree of freedom, Kelvin's theorem is modified

$$\Omega = b + \omega + \frac{s}{2}\sqrt{G}R[G] \qquad \dot{\Omega} + \vec{\nabla}\cdot(\Omega\vec{v}) = 0$$

$$\rho_e = \frac{B}{4\pi} - \frac{b+\omega}{4\pi}$$

- Ω = constant

$$\delta\rho_{\rm e} = \frac{s}{8\pi}\sqrt{G}\,R[G].$$

An immediate consequence

$$\delta\rho_e = \frac{s}{8\pi}\sqrt{G}R[G] \quad \sim \partial_i \partial_j G_{ij}$$

$$\to \langle \delta\rho_e \delta\rho_e \rangle_{\omega,q} \sim q^4$$

Property of the lowest Landau level

- Since R ~ ∂∂G: density-density correlation functions ~ q^4
- The static structure factor is independent of H

$$G_{ij} = \delta_{ij} + h_{ij}$$

$$\langle \delta\rho_{\rm e} \delta\rho_{\rm e} \rangle_q = \frac{N(N+1)}{2N+1} \frac{q^4}{16\pi B} \, .$$

- Since R ~ ∂∂G: density-density correlation functions ~ q^4
- The static structure factor is independent of H

$$G_{ij} = \delta_{ij} + h_{ij}$$

$$[h_{\bar{z}\bar{z}}(\mathbf{x},\, h_{zz}(\mathbf{y})] = \frac{1}{s}\delta(\mathbf{x}-\mathbf{y})$$

$$\langle \delta\rho_{\rm e}\delta\rho_{\rm e}\rangle_q = \frac{N(N+1)}{2N+1}\frac{q^4}{16\pi B}\,.$$

- Since R ~ ∂∂G: density-density correlation functions ~ q^4
- The static structure factor is independent of H

$$G_{ij} = \delta_{ij} + h_{ij}$$

$$[h_{\bar{z}\bar{z}}(\mathbf{x},\, h_{zz}(\mathbf{y})] = \frac{1}{s}\delta(\mathbf{x}-\mathbf{y})$$

$$a \qquad a^\dagger$$

$$\langle \delta\rho_{\mathrm{e}}\delta\rho_{\mathrm{e}}\rangle_q = \frac{N(N+1)}{2N+1}\frac{q^4}{16\pi B}\,.$$

- Since R ~ ∂∂G: density-density correlation functions ~ q^4
- The static structure factor is independent of H

$$G_{ij} = \delta_{ij} + h_{ij} \qquad [h_{\bar{z}\bar{z}}(\mathbf{x},\, h_{zz}(\mathbf{y})] = \frac{1}{s}\delta(\mathbf{x}-\mathbf{y})$$

$$a \qquad a^\dagger$$

$$\delta\rho_{\mathrm{el}} \sim \partial_z^2 h_{\bar{z}\bar{z}} + \partial_{\bar{z}}^2 h_{zz}$$

$$\boxed{\langle \delta\rho_{\mathrm{e}}\delta\rho_{\mathrm{e}}\rangle_q = \frac{N(N+1)}{2N+1}\frac{q^4}{16\pi B}\,.}$$

- Since R ~ ∂∂G: density-density correlation functions ~ q^4
- The static structure factor is independent of H

$$G_{ij} = \delta_{ij} + h_{ij}$$

$$[h_{\bar{z}\bar{z}}(\mathbf{x},\, h_{zz}(\mathbf{y})] = \frac{1}{s}\delta(\mathbf{x}-\mathbf{y})$$

a $\quad$ $a^\dagger$

$$\delta\rho_{\rm el} \sim \partial_z^2 h_{\bar{z}\bar{z}} + \partial_{\bar{z}}^2 h_{zz}$$

$$\langle \delta\rho_{\rm e}\delta\rho_{\rm e}\rangle_q = \frac{N(N+1)}{2N+1}\frac{q^4}{16\pi B}\,.$$

should be good for large *N*

for *N*=1 (nu=1/3, 2/3): reproduces exact value from Laughlin wf

- Since R ~ ∂∂G: density-density correlation functions ~ q^4
- The static structure factor is independent of H

$$G_{ij} = \delta_{ij} + h_{ij} \qquad [h_{\bar z\bar z}(\mathbf{x},\, h_{zz}(\mathbf{y})] = \frac{1}{s}\delta(\mathbf{x}-\mathbf{y})$$

a $\qquad a^\dagger$

$$\delta\rho_{\rm el} \sim \partial_z^2 h_{\bar z\bar z} + \partial_{\bar z}^2 h_{zz}$$

$$\langle \delta\rho_{\rm e}\delta\rho_{\rm e}\rangle_q = \frac{N(N+1)}{2N+1}\frac{q^4}{16\pi B}\,.$$

should be good for large *N*
for *N*=1 (nu=1/3, 2/3): reproduces exact value from Laughlin wf

(HLR: 1/2 or 2 times Laughlin's value)

Conclusion

- Low-q regime of FQH liquid: described by a fluid with internal metric degree of freedom, coupled to a gauge field
- Electron density ~ curvature of dynamic metric
- Static structure factor: algebraic calculation

Thank you