Taming Quantum Entanglement

Simons – Ultra Quantrum Matter Conf
Boston 9/13/19

MPA Fisher

- Classical system: Entropy always increases (2nd law of thermo)
- Isolated Quantum system: Entanglement entropy (= thermal entropy)
- Entanglement entropy always grows

"Disorder always reigns"

How to control (entanglement) entropy growth?

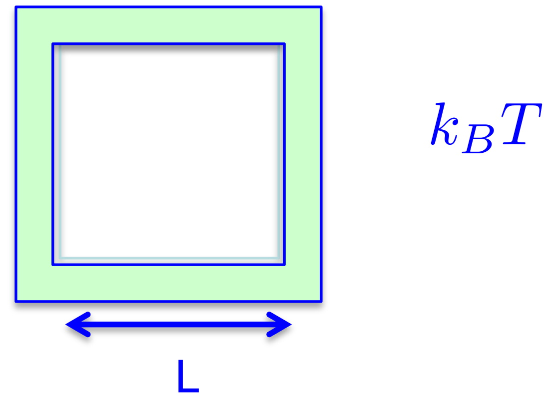
1. "Many-body localization" **-** Localization from disorder
2. "Quantum disentangled states" **-** Heavy particles "localizing" light particles
3. **Measurement driven entanglement transition**

Entropy: Thermal "versus" entanglement

Thermal entropy:

Number of states, extensive for T>0

$$S_{th} = -Tr[\hat{\rho}_{th} \ln \hat{\rho}_{th}] \sim L^d$$

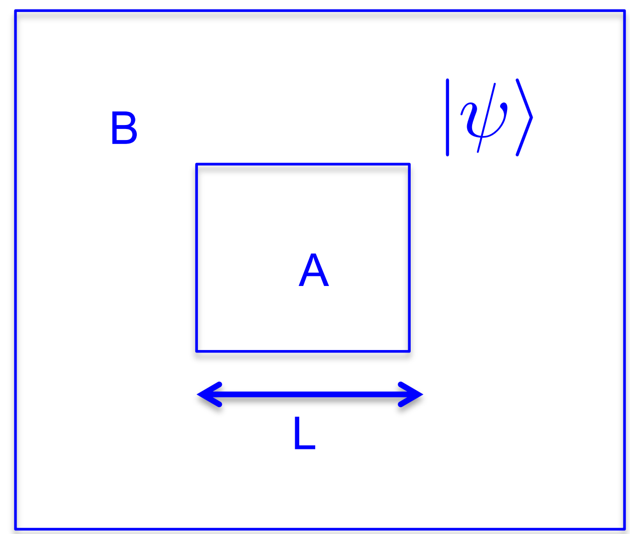


Entanglement Entropy: Single eigenstate

$$\hat{\mathcal{H}}|\psi\rangle = E|\psi\rangle$$
$$\hat{\rho} = |\psi\rangle\langle\psi|$$
$$\hat{\rho}_A = Tr_B(\hat{\rho})$$

Entanglement entropy:

$$S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A)$$



ETH: Equivalence of Thermal and entanglement entropies

$$S_A/L^d = S_{th}/L^d; \quad L \to \infty$$

Thermal entropy is state counting, entanglement entropy depends on the properties of the states!

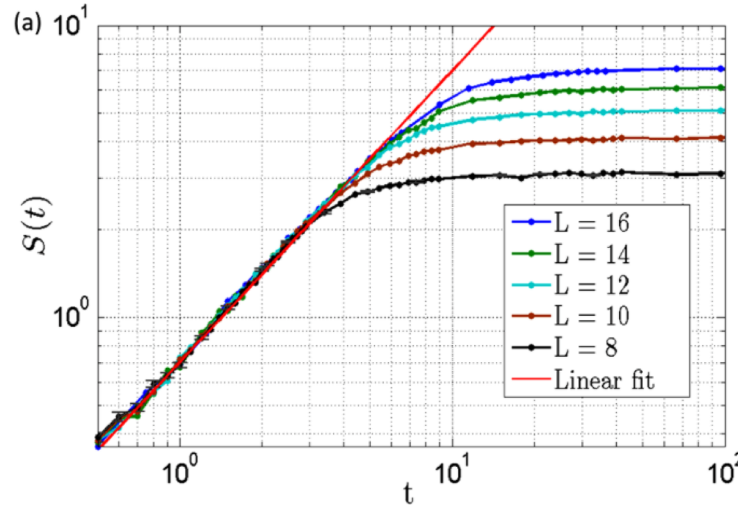
Entanglement Dynamics (i.e. Growth)

1) Quantum Quench

Evolve unentangled initial state w/ Hamiltonian

$$H = \sum_{i=1}^{L} \left(g\sigma_i^x + h\sigma_i^z + J\sigma_i^z \sigma_{i+1}^z \right)$$

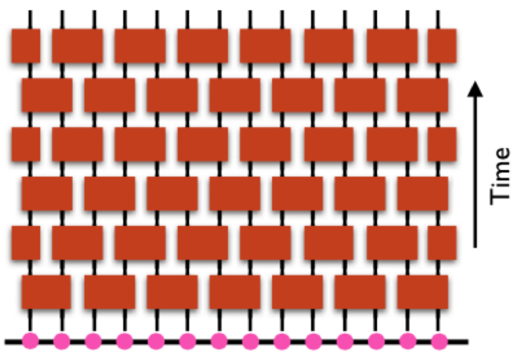
Entanglement spreads ballistically,
even though energy diffuses

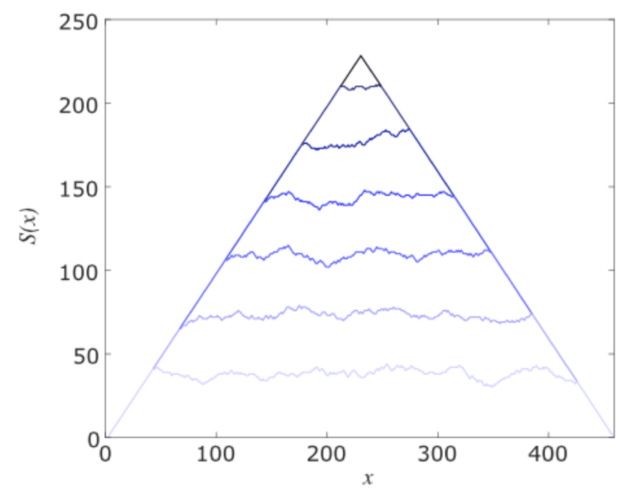


Half-cut entanglement entropy Kim + Huse (2013)

2) Unitary Dynamics with no energy conservation

Quantum circuit: evolve Qubits w/ (random) unitary gates
Initial state: unentangled product state
Entanglement spreads ballistically, into maximal entropy state

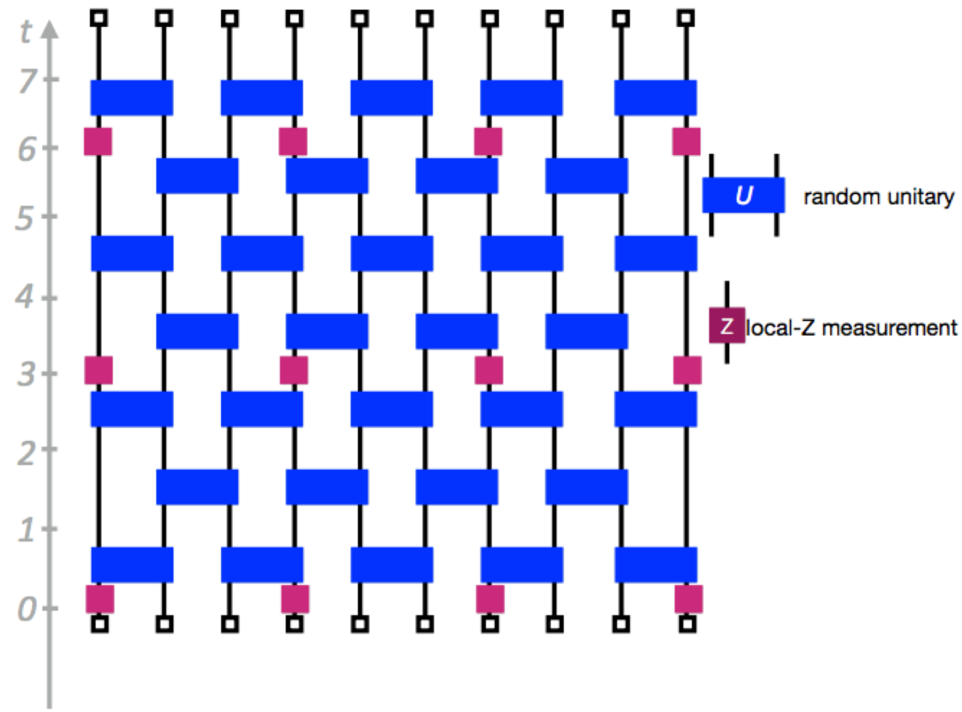




Nahum, Ruhman, Vijay, Haah (2017)

How to control (entanglement) entropy growth?

Via Measurements



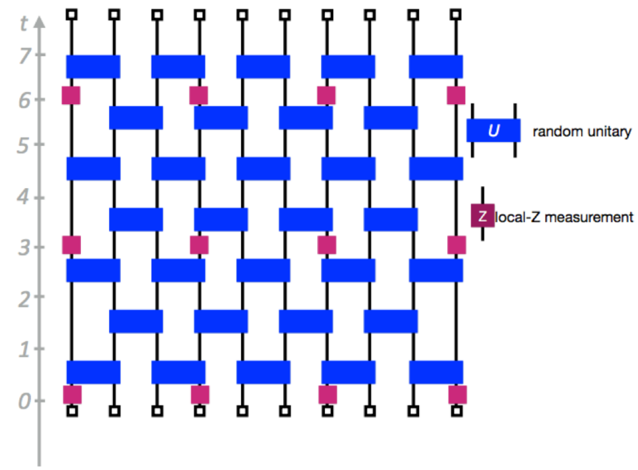
Measurement driven entanglement transition

Taming entanglement w/ ***measurements***

“Hybrid Quantum Circuit” w/ both unitary and measurement gates

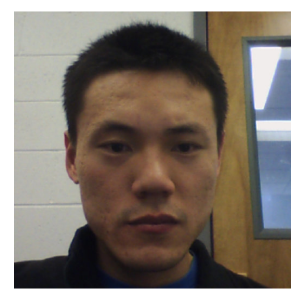
- Unitary evolution induces entanglement growth
- Measurements induce disentanglement

Explore competition between unitary evolution and measurements



- Li, Chen, MPAF (2018/2019)
- Skinner, Ruhman, Nahum (2018)
- Chan, Nandkishore, Pretko, Smith (2018)
- Choi, Bao, Qi, Altman (2019)
- Gullans, Huse (2019)

Yaodong Li



Xiao Chen

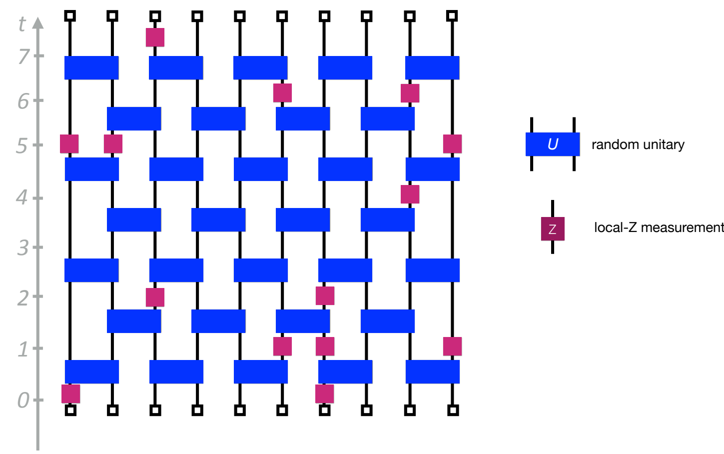
“Hybrid” Quantum Circuit

Quantum circuit w/ unitary gates and projective measurements

2-Qubit Unitaries:

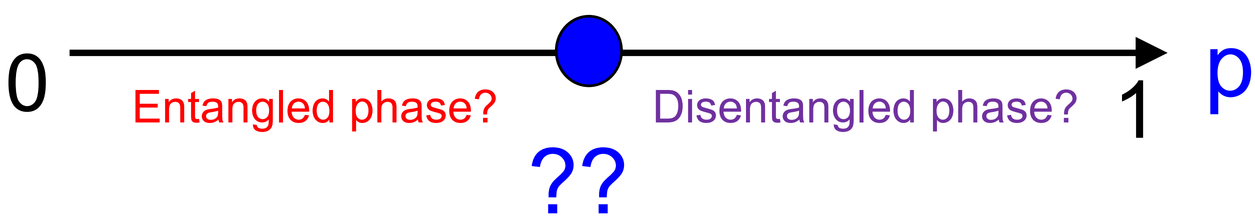
1-Qubit Measurements

Make measurements with probability, p

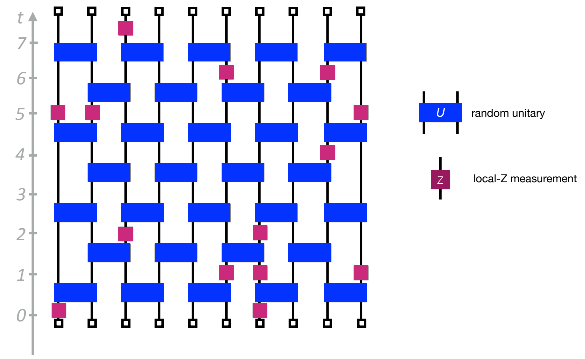


Phase Diagram??

- p=0; No measurement, Volume law entanglement
- p=1; Measure every Qubit, no entanglement (area law)
- Transition at $p=p_c$??



Numerics on Hybrid Circuits?



Direct simulation very challenging for large L
(since the Hilbert space grows as 2^{L})

Employ Quantum information "technology":

- **"Stabilizers"** to encode special **"codeword"** quantum states
- Evolve stabilizers with **Clifford unitaries**
- Measurements of Z-component of spin

Gottesman-Knill Theorem: Such quantum circuits can be efficiently simulated on a classical computer (accessing >500 Qubits, say)

Pauli Strings, Stabilizers and Codewords

Pauli operators for a single Qubit $\{1, \sigma_x, \sigma_y, \sigma_z\} \rightarrow \{1, X, Y, Z\}$

Pauli String Operators for L Qubits: $g = 1_1 Y_2 X_3 I_4 X_5 ... Z_L$

g

Stabilizers and “codewords”:

$|\psi\rangle$ is a “codeword” state if “stabilized” by L independent, commuting Pauli string operators $g_j|\psi\rangle = |\psi\rangle$

Example 1: $|\psi\rangle = |00, ...0\rangle$ is stabilized by $g_j = Z_j$

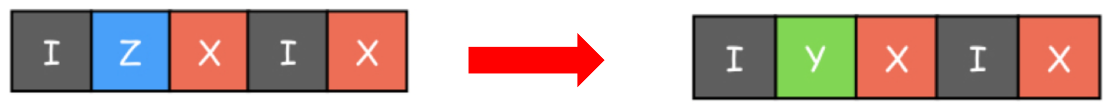
Example 2: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is stabilized by $g_1 = Z_1 Z_2$

$g_2 = X_1 X_2$

Clifford Unitaries/Dynamics

Clifford unitaries take Pauli string operators into other Pauli string operators

$$\hat{U}\hat{g}U^{\dagger} = \hat{g}'$$



Unitary evolution of a "codeword" state: follow the dynamics of the L stabilizers:

If $|\psi\rangle$ stabilized by g_j then $|\psi'\rangle = U|\psi\rangle$ stabilized by $g_j' = U g_j U^{\dagger}$

Measurements and Stabilizers

Consider a projective measurement of a codeword $g_j|\psi\rangle = |\psi\rangle$

$$|\psi\rangle \rightarrow P_{\pm}|\psi\rangle \qquad P_{\pm} = (1 \pm Z_j)/2$$

Measuring Z-component of j^{th} qubit

If Z_j anticommutes with g_1 and commutes with $g_2,...,g_L$ (say)
the stabilizers are modified under the measurement as:

$$\{g_1, g_2, ..., g_L\} \rightarrow \{\pm Z_j, g_2, ..., g_L\}$$

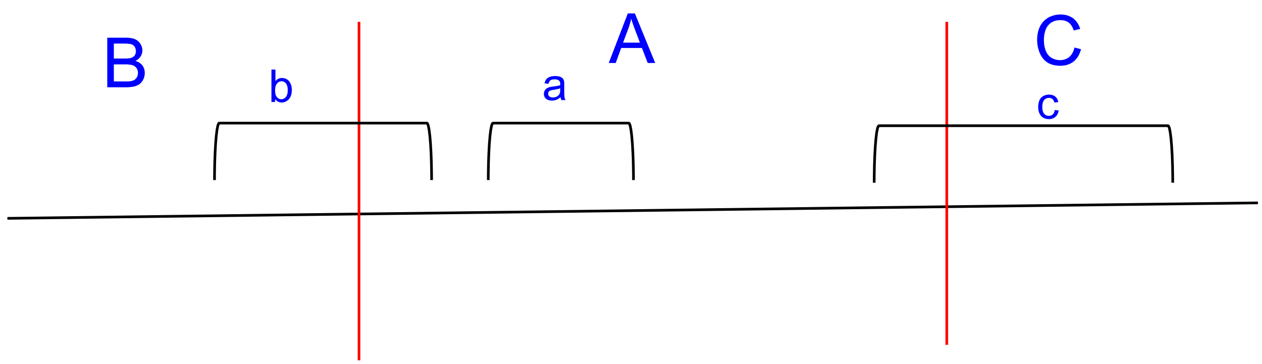
when the “result” of the measurement is ± 1

Entanglement and Stabilizers

Stabilizer length length=6

$$g = 1_1 1_2 X_3 1_4 Z_5 Y_6 1_7 Z_8 1_9 1_{10}$$

Entanglement entropy S_A



Denote number of stabilizers starting in A and ending in A,B,C as n_a, n_b, n_c

Entanglement: $$S_A = \frac{(n_b + n_c)}{2} \log(2)$$

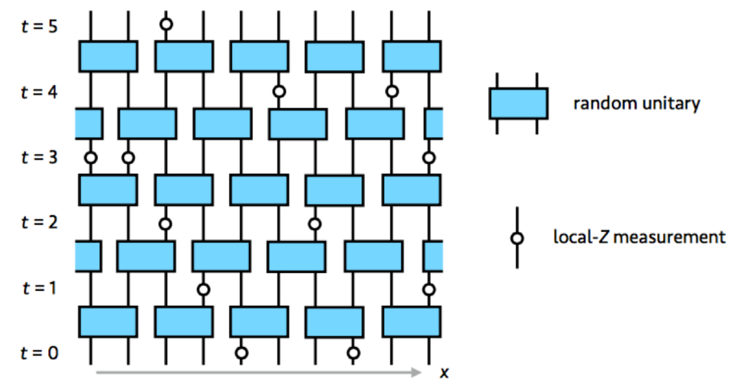
Clifford Circuit: Simulable

All 2-Qubit unitaries taken from the Clifford group:

$$|\psi_t\rangle \to |\psi_{t+1}\rangle = U|\psi_t\rangle$$

All single Qubit measurements taken from Pauli group

$$|\psi\rangle \to \frac{P_\pm |\psi\rangle}{\sqrt{p_\pm}} \qquad P_\pm = \frac{1}{2}(1 \pm Z)$$



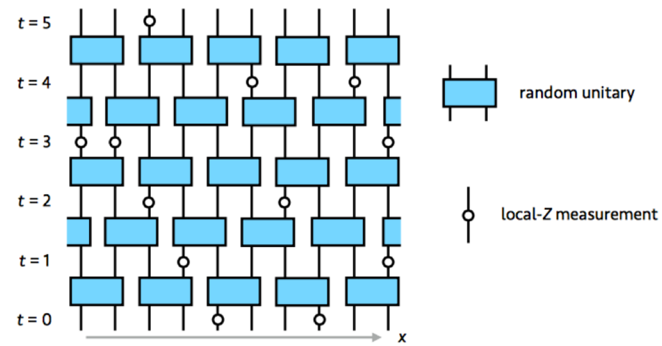
Make measurements with probability p

Simulate Clifford quantum circuits on classical computer
(accessing >500 Qubits)

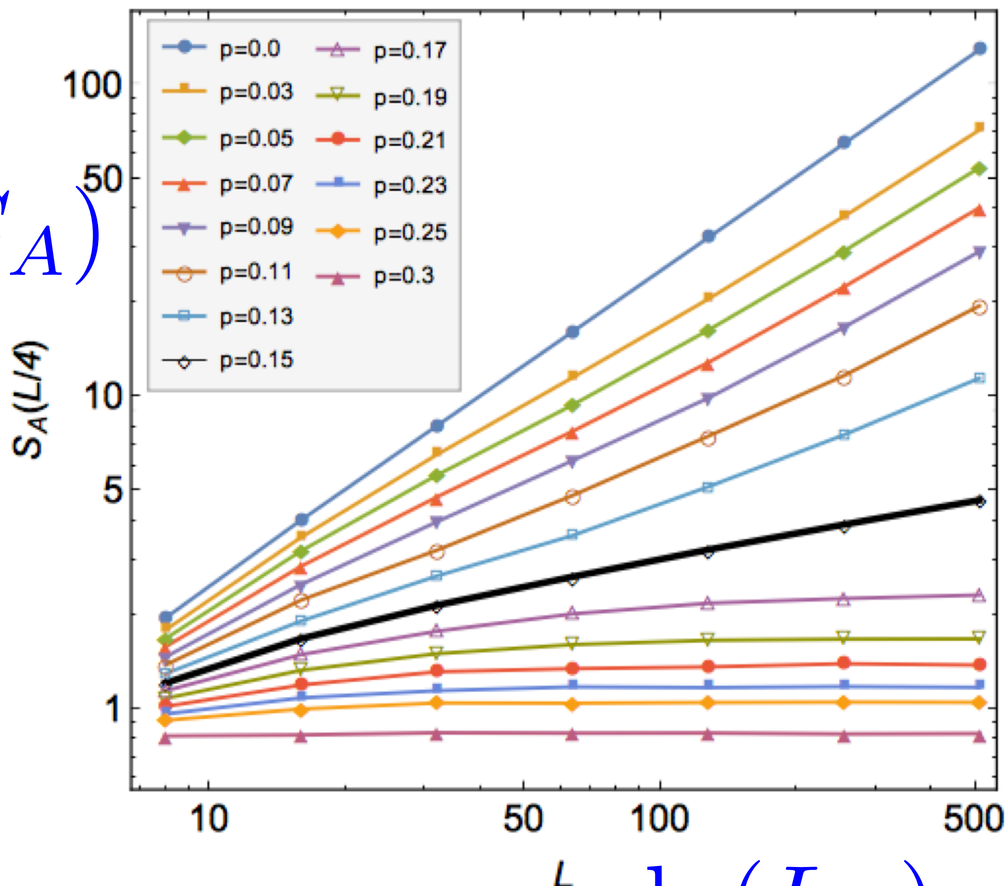
(Comment: For Clifford circuits, all Renyi entropies are equal)

Entanglement Entropy

Long-time steady-state of Clifford circuit



$\ln(S_A)$



$\ln(L_A)$

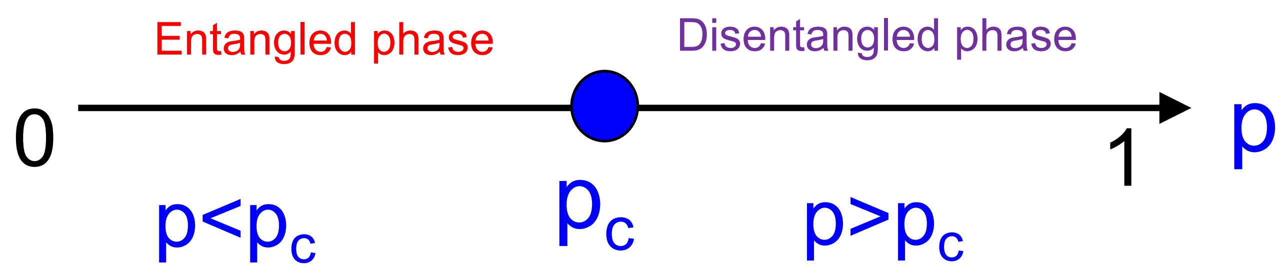
Volume law entanglement

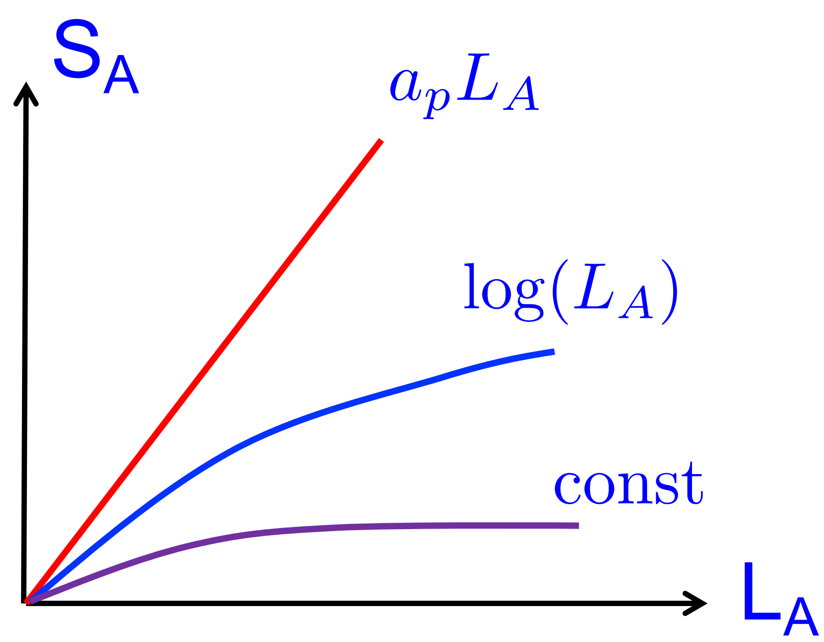
Increasing measurement rate

Area law entanglement

Entanglement Transition

Li, Chen, MPAF (2018)

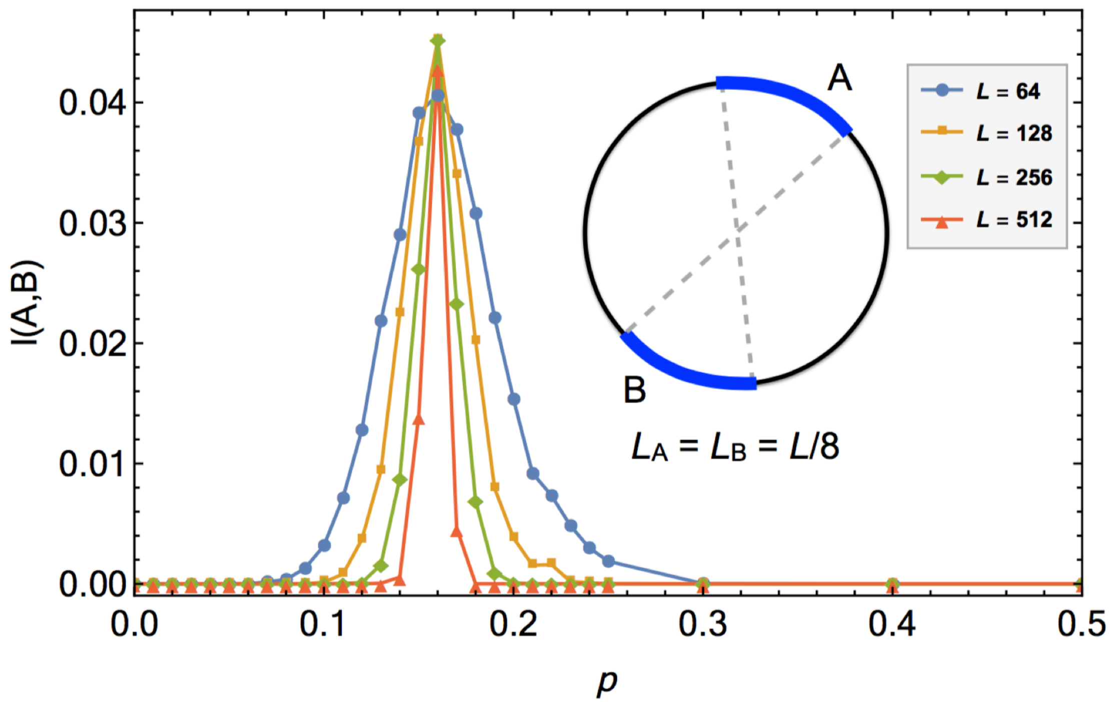




$$S_A(L_A) \sim \begin{cases} a_p L_A; & p < p_c \\ \log(L_A); & p = p_c \\ \text{const}; & p > p_c \end{cases}$$

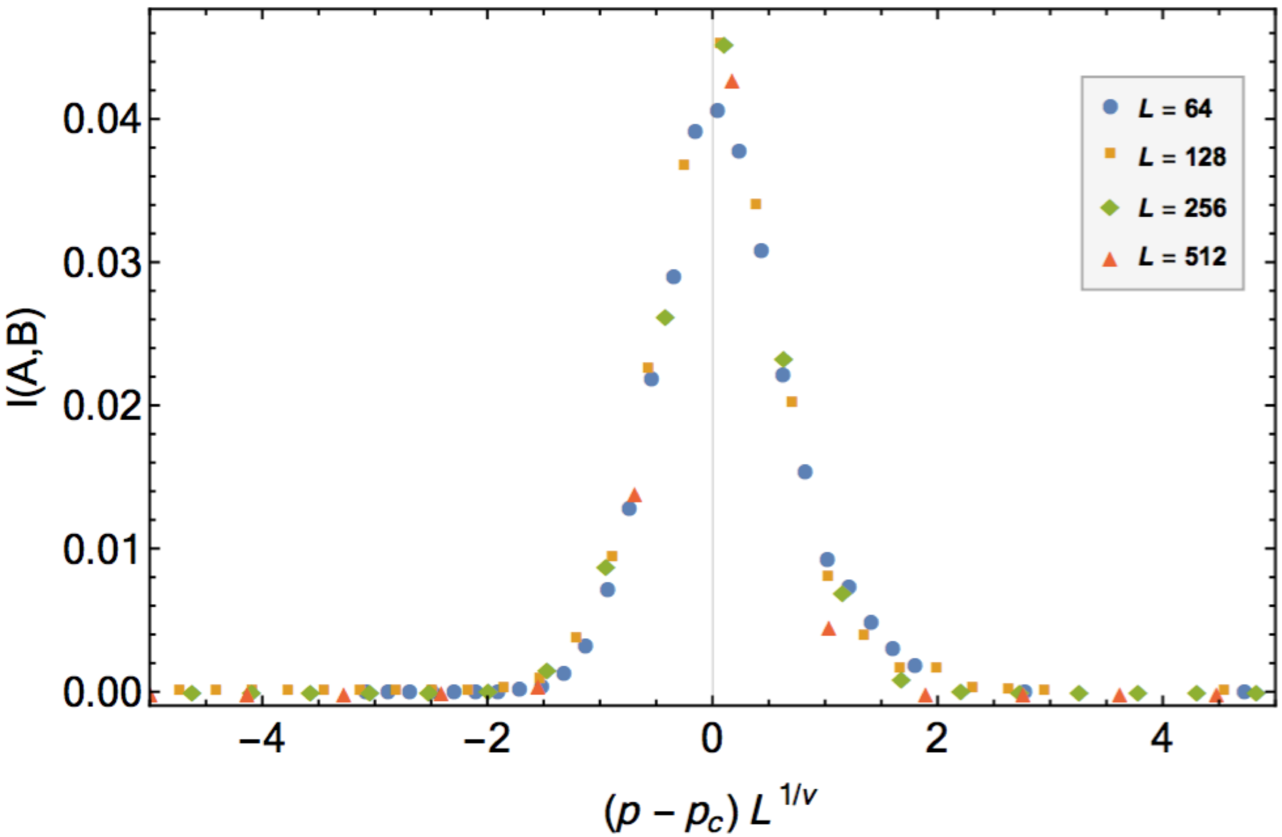
Mutual Information: Locates transition

$$\mathcal{I}_{AB} = S_A + S_B - S_{AB}$$



$$\mathcal{I}_{AB}(L \to \infty) = \begin{cases} 0; & p \neq p_c \\ \text{const}; & p = p_c \end{cases}$$

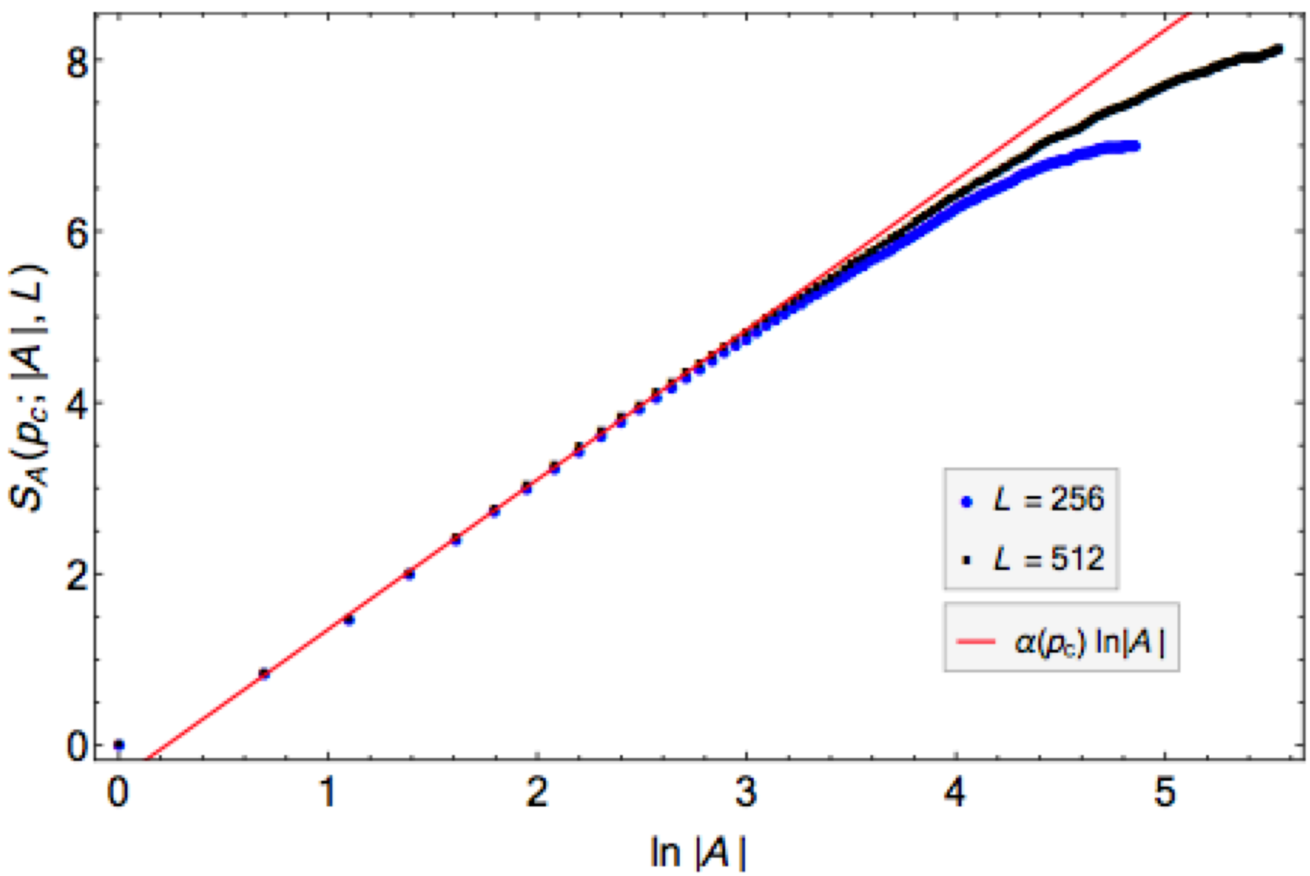
Data Collapse: Mutual Information



$$\nu \approx 1.4$$

Log Scaling at Criticality ($p=p_c$)

$$S_A(L_A) = \alpha_c \log(L_A) \qquad \alpha_c \approx 1.6$$

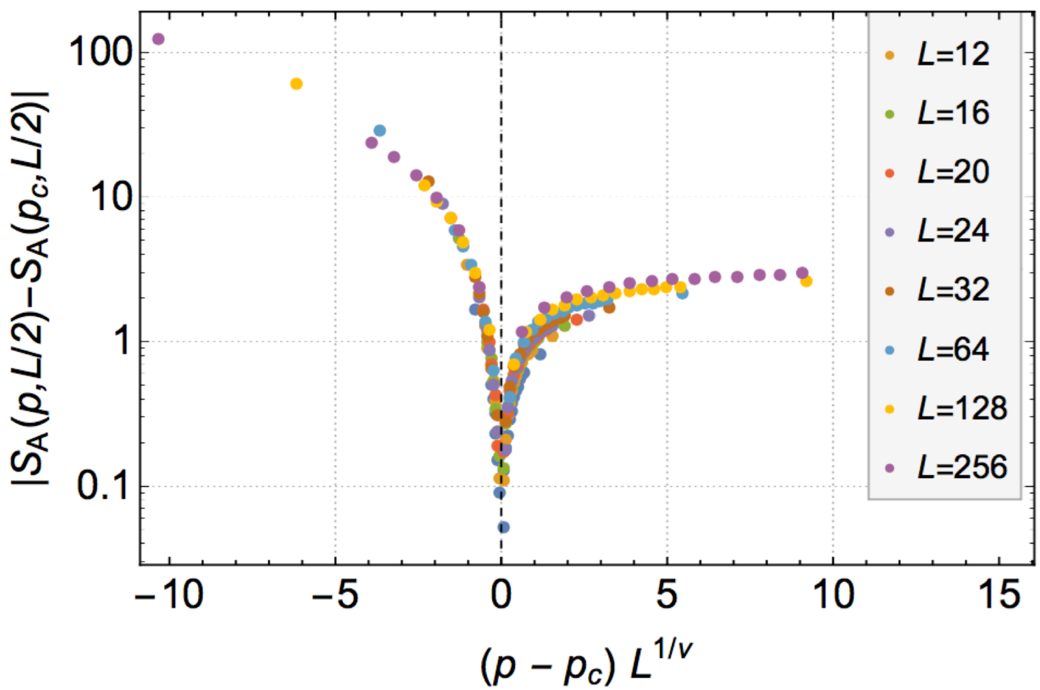


"Log" Scaling Collapse

$$S_A(p, L_A) = A \log L_A + G(L_A/\xi)$$

$$\xi \sim |p - p_c|^{-\nu} \qquad \nu \approx 1.4$$

$$S_A(p, L_A) - S_A(p_c, L_A) = \tilde{G}(L_A/\xi)$$



Conformal Symmetry at criticality ($p=p_c$)

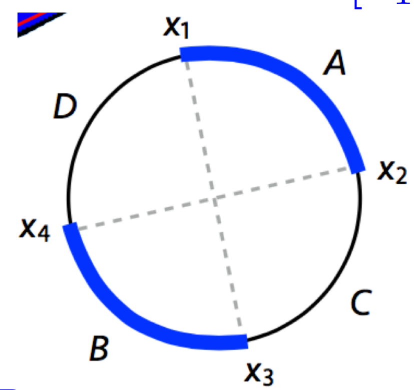
If have underlying conformal field theory, then mutual information depends only on the cross ratio

$$I_{AB} = f(\eta)$$

$$\eta \equiv \frac{x_{12}x_{34}}{x_{13}x_{24}}$$

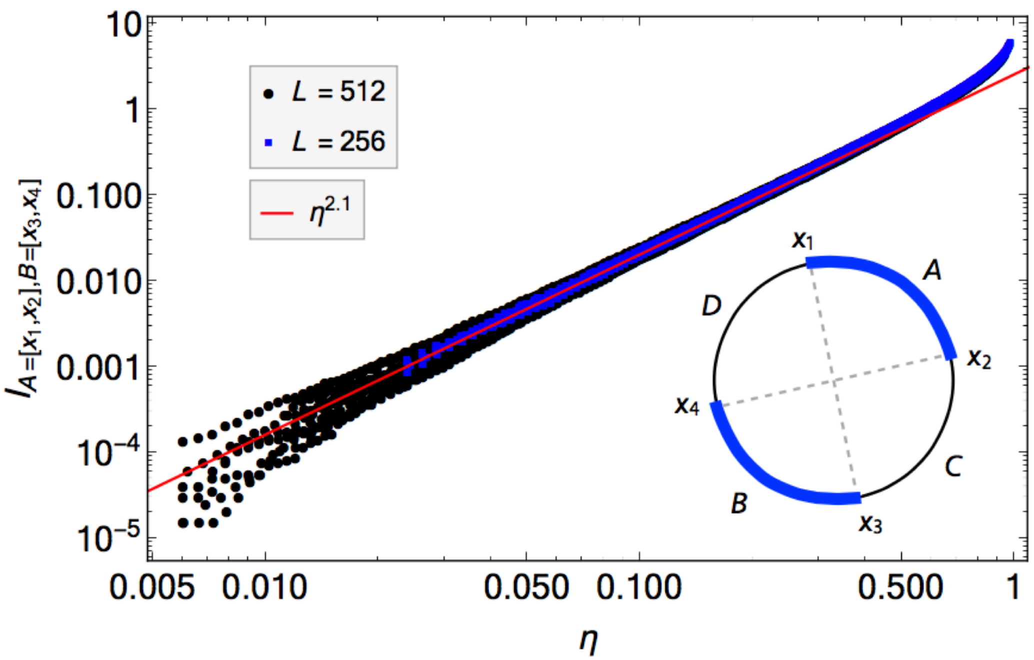
$$x_{ij} = \frac{L}{\pi}\sin\left(\frac{\pi}{L}|x_i - x_j|\right)$$

$A = [x_1, x_2]$



$B = [x_3, x_4]$

I_{AB}



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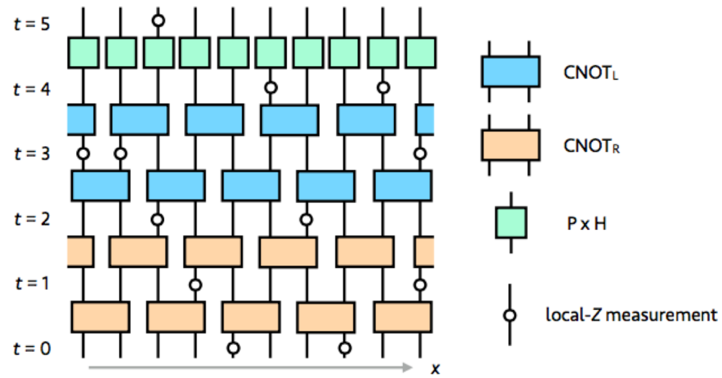
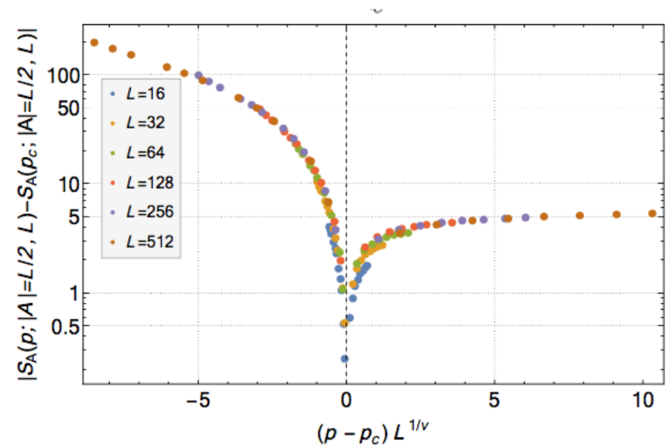
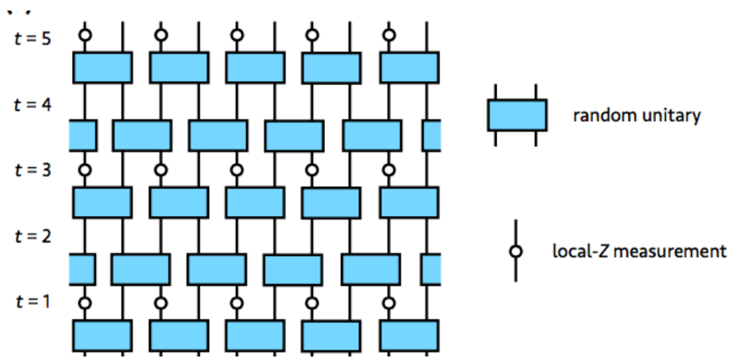
Circuits with (Translational) Symmetry

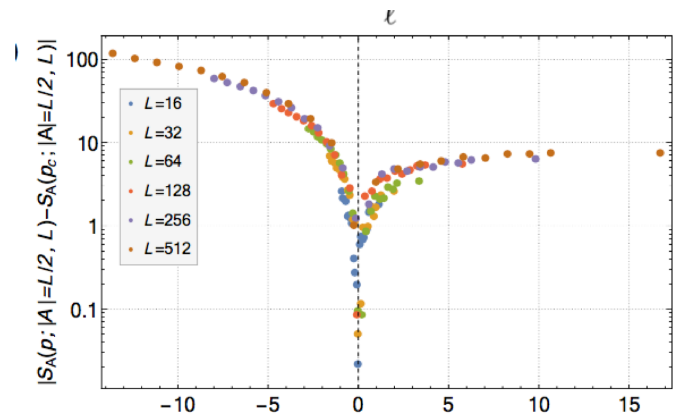
Floquet-Clifford Circuit



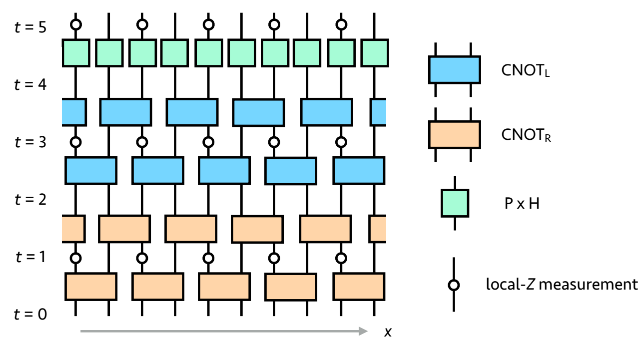


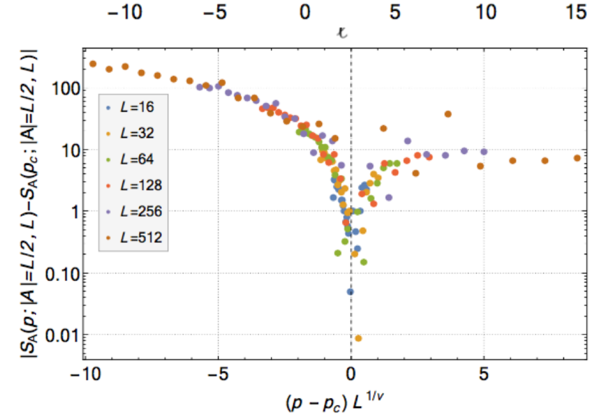
Circuit with (quasi-) periodic measurement locations





Floquet w/ periodic measurements (no randomness)





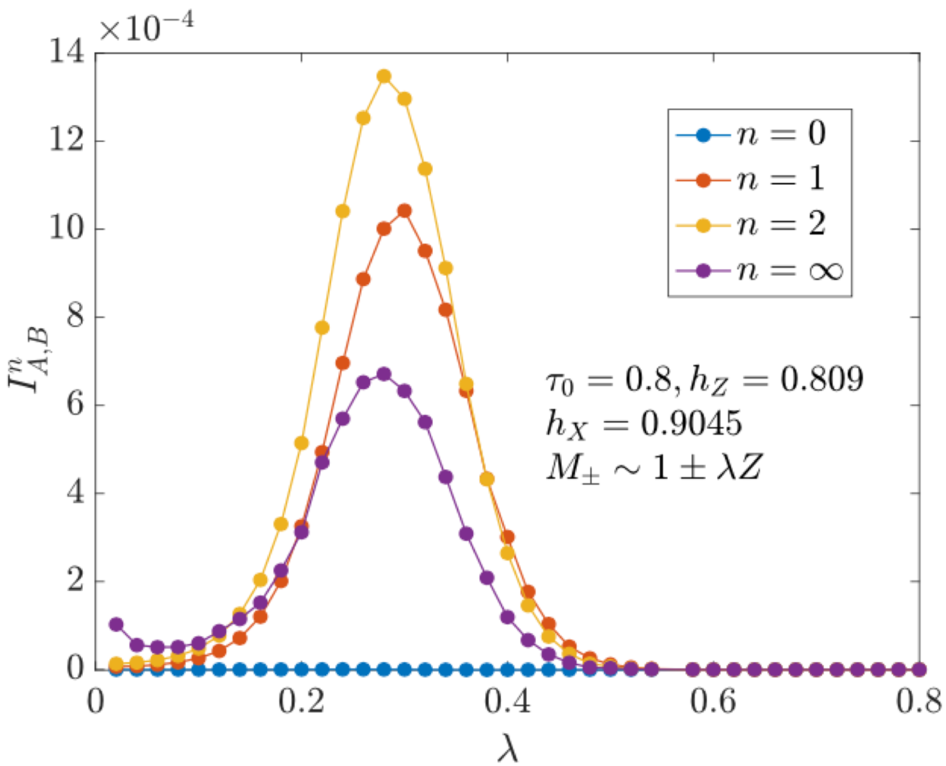
All exponents the same!!

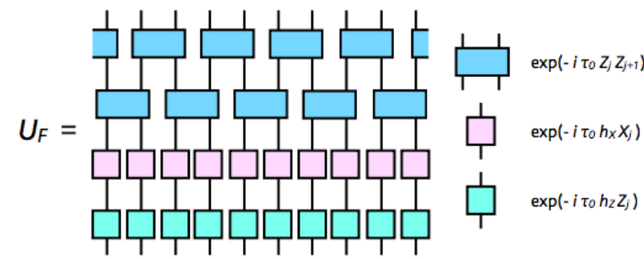
Beyond Clifford: “Ising” Floquet Unitaries

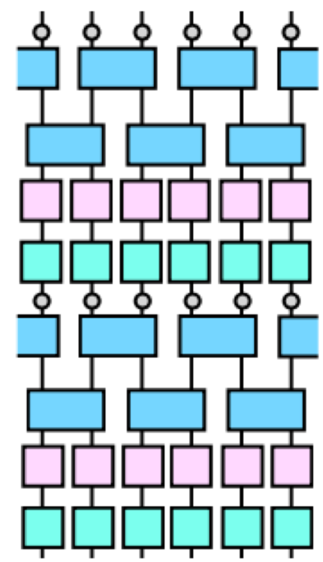
With generalized measurements: $M_{\pm} = \frac{1 \pm \lambda Z}{\sqrt{2(1+\lambda^2)}}$ $\lambda \in [0, 1]$

No randomness

$$I^n_{AB} = S^n_A + S^n_B - S^n_{AB}$$







Purification Transition induced by measurements

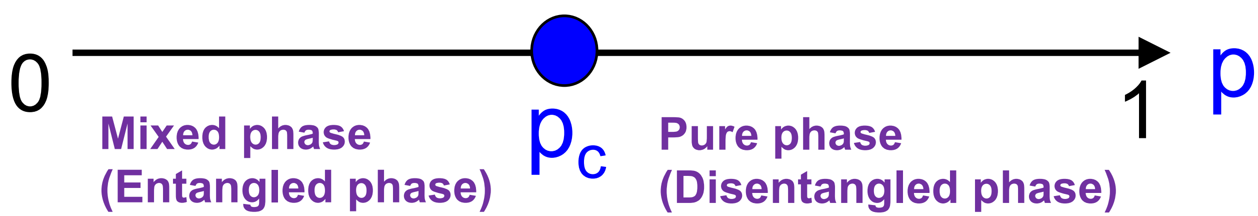
Gullans, Huse (2019)

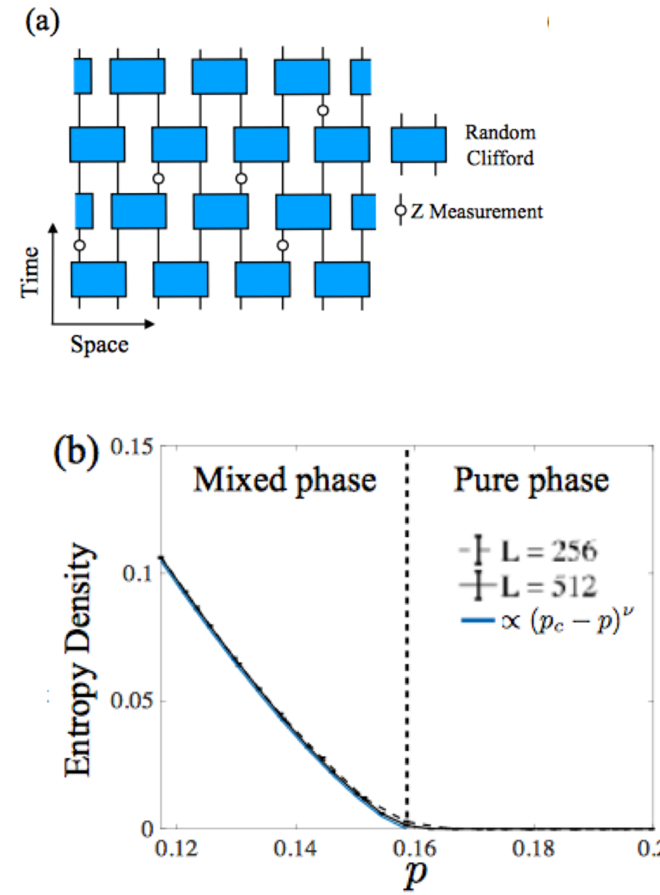
Start dynamics in maximally mixed state, run for $t \approx cL$

$$\hat{\rho}(t=0) = \frac{1}{2^L}\hat{1}$$

Compute averaged thermal entropy

$$\langle S(\rho) \rangle = -Tr(\rho \ln \rho)$$



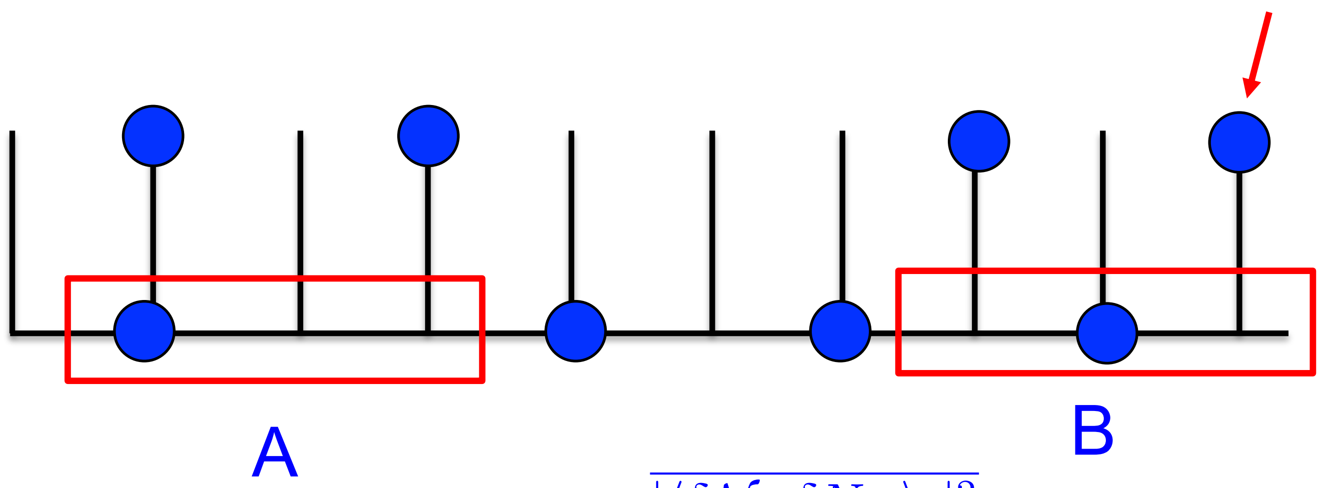


Purification transition = entanglement transition

Experimental Access??

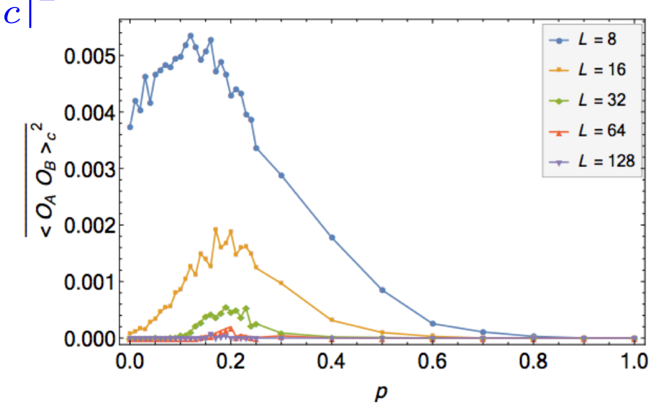
Particles on “Comb” Lattice

Make projective measurements (Z_x)



Peak in correlation functions at criticality (as in Clifford)

$\overline{|\langle \delta \mathcal{N}_A \delta N_B \rangle_c|^2}$



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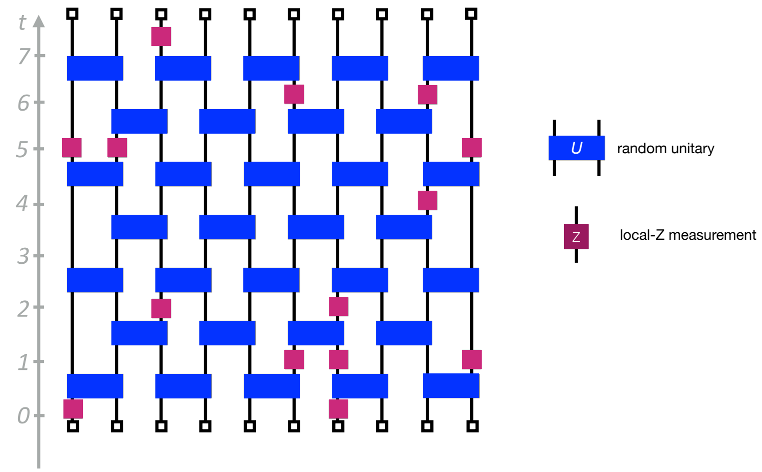
But in practice? Hard to measure from ensemble of (different) pure states

$$\langle \psi | \delta \mathcal{N}_A \delta N_B | \psi \rangle$$

Why? Cannot measure expectation value in a one-shot measurement

Summary: Taming Entanglement

Quantum Entanglement Transition:



Competition between unitary induced entanglement and measurement induced disentanglement

(Entanglement transition = purification transition)

Open/future:

- Entanglement transition in d=1;
  - ➢ Space-time conformal symmetry in 1+1?
  - ➢ Analytic access to (non-unitary) CFT?
  - ➢ Genericity of Clifford dynamics?
- Transitions in d>1?
- Experimental access?? Quantum computer??

Correlation functions

Mutual information upper bound for all correlation functions

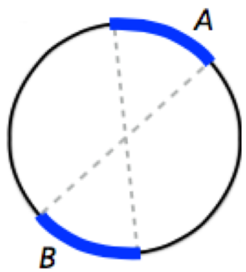
$$I_{AB} \geq \frac{1}{2} \frac{|\langle \mathcal{O}_A \mathcal{O}_B \rangle_c|^2}{||\mathcal{O}_A||^2 ||\mathcal{O}_B||^2}$$

Averaged squared correlation function (not equal to expectation value of any operator)

$$\overline{|\langle \mathcal{O}_A \mathcal{O}_B \rangle_c|^2} \neq Tr(\rho \mathcal{O}_{A \cup B})$$

$$\mathcal{O}_A = \sum_{x \in A} Z_x$$

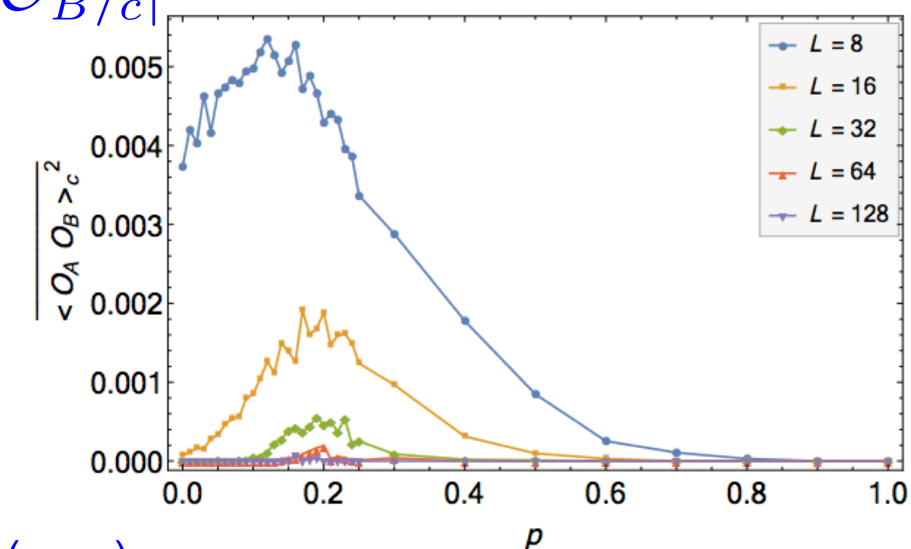
$$\mathcal{O}_B = \sum_{x \in B} Z_x$$



|A| = |B| = L/8

$$\overline{|\langle \mathcal{O}_A \mathcal{O}_B \rangle_c|^2}$$

Peak at $p = p_c$



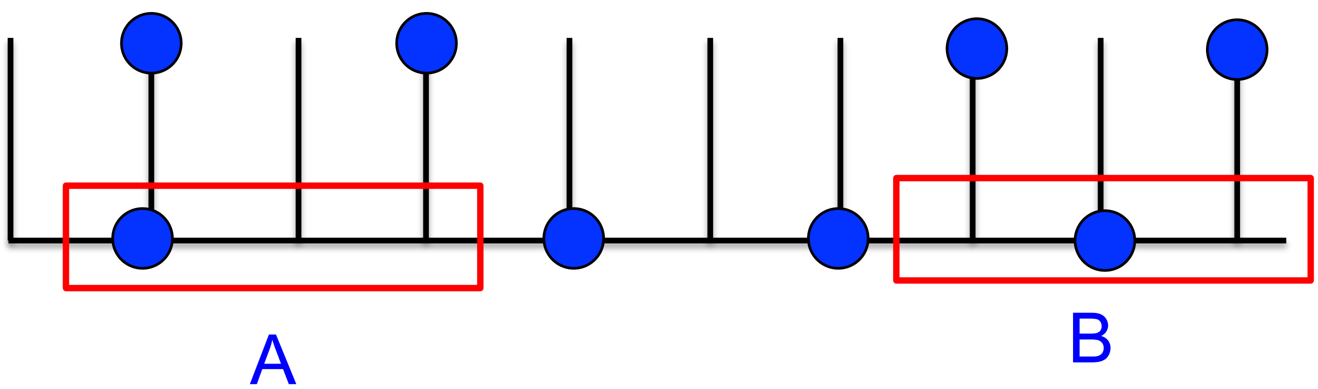
$$p$$

Consistent w/ power law decay at criticality ($p = p_c$)

$$\overline{|\langle \mathcal{O}_A \mathcal{O}_B \rangle_c|^2} \sim |x_A - x_B|^{-\gamma}$$

Cold/Rydberg Atoms?

"Comb" Lattice



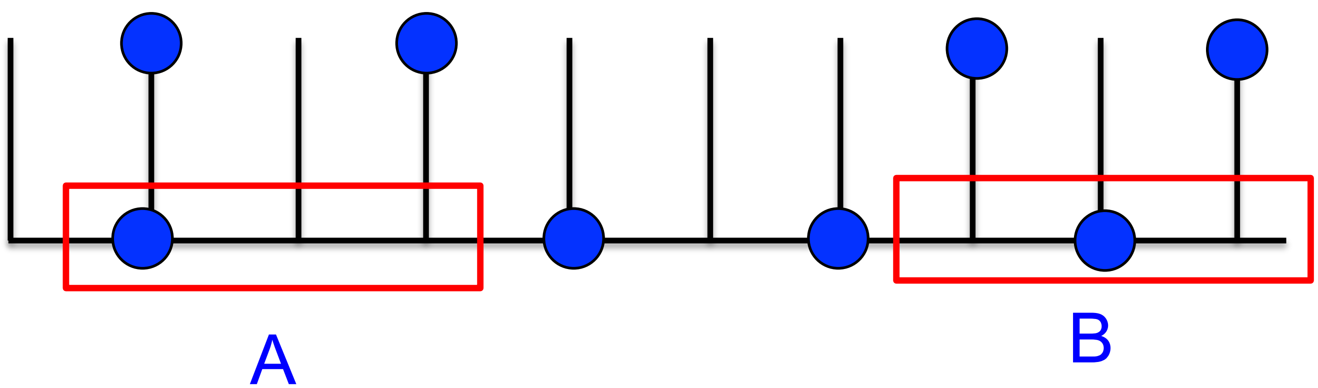
Optical microscope:

- Bosons hopping on a "comb" lattice
- Make projective measurements of boson number on "top" of "teeth"
- Compute/measure I_{AB} and squared number fluctuation correlation function
- Expect power law decay at criticality

$$\overline{|\langle \delta \mathcal{N}_A \delta N_B \rangle_c|^2} \sim |x_A - x_B|^{-\gamma}$$

Cold/Rydberg Atoms?

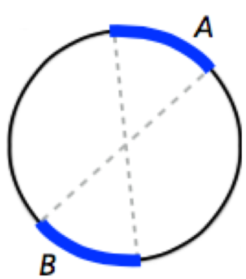
"Comb" Lattice



Expect peak in correlation functions at criticality (as in Clifford)

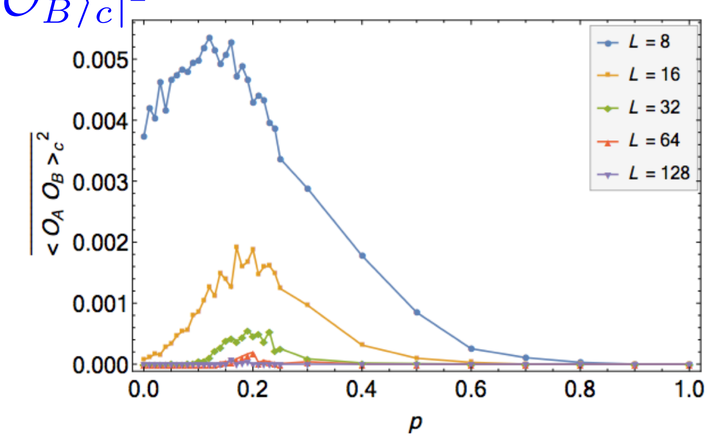
$$\mathcal{O}_A = \sum_{x \in A} Z_x$$

$$\mathcal{O}_B = \sum_{x \in B} Z_x$$



|A| = |B| = L/8

$\overline{|\langle \mathcal{O}_A \mathcal{O}_B \rangle_c|^2}$



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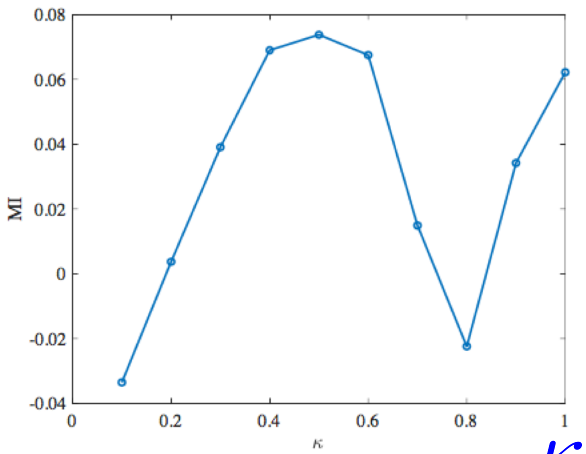
Transition accessible in principle

See a peak for L=20, L_A=1, L_B =1, $x_A – x_B$ = 10

I_{AB}



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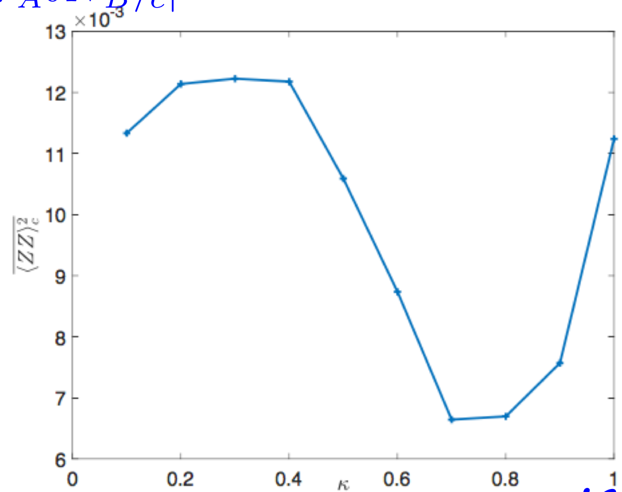
$\overline{|\langle \delta \mathcal{N}_A \delta N_B \rangle_c|^2}$



κ

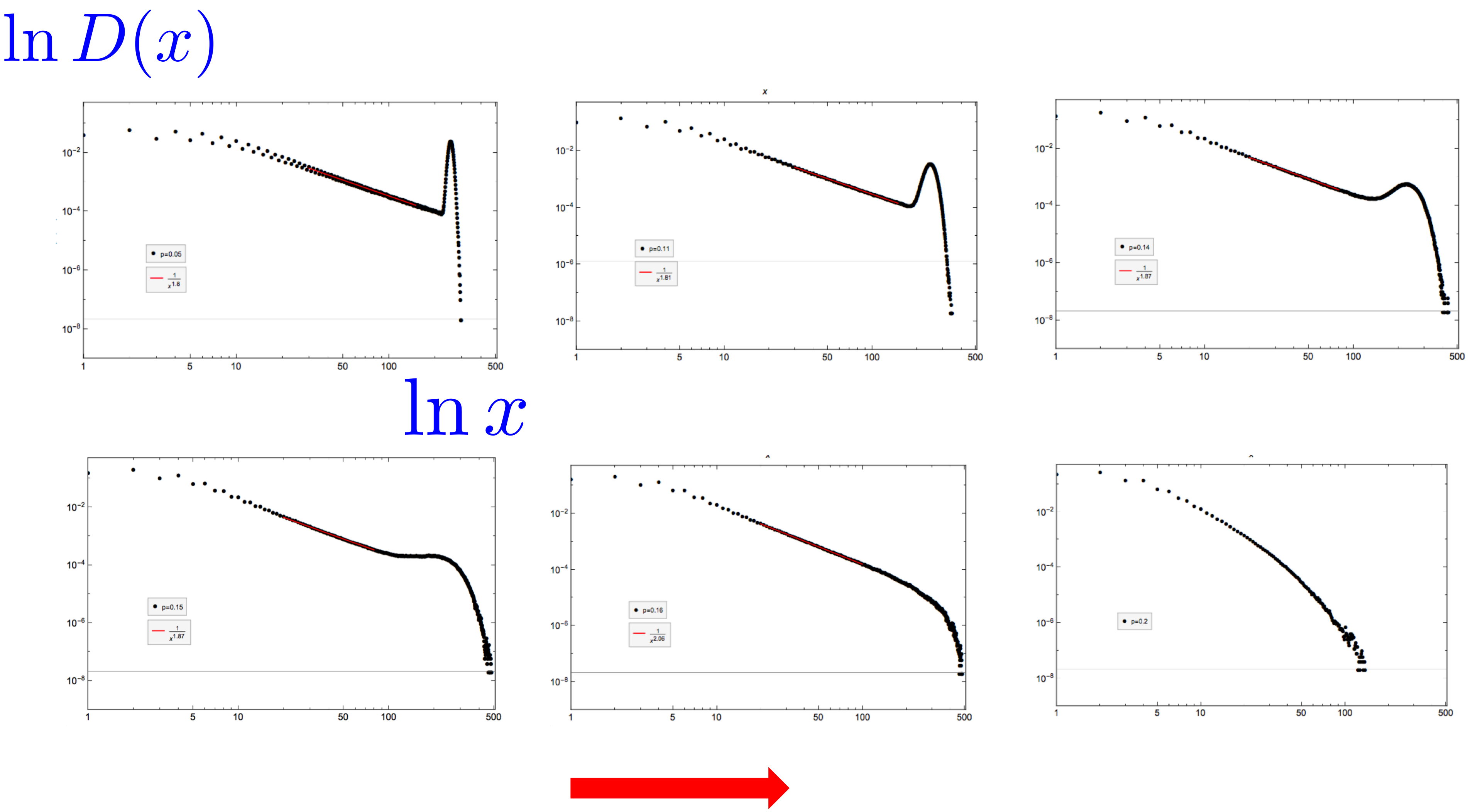
But in practice? Might be hard to measure from ensemble of (different) pure states

$$\langle \psi | \delta \mathcal{N}_A \delta N_B | \psi \rangle$$

Why?

- Cannot measure expectation value in a one-shot measurement (exploit self-averaging?)
- Making multiple copies of each pure state will be hard

"Hidden log" inside volume law phase: Stabilizer length distribution function



Increasing measurement rate

Stabilizer length distribution function

$$D(x,L) \approx \begin{cases} \frac{b_p}{x^2} + a_p\delta(x - L/2); & p < p_c \\ \frac{b_c}{x^2}; & p = p_c \\ \frac{e^{-x/\xi}}{x^2}; & p > p_c \end{cases}$$

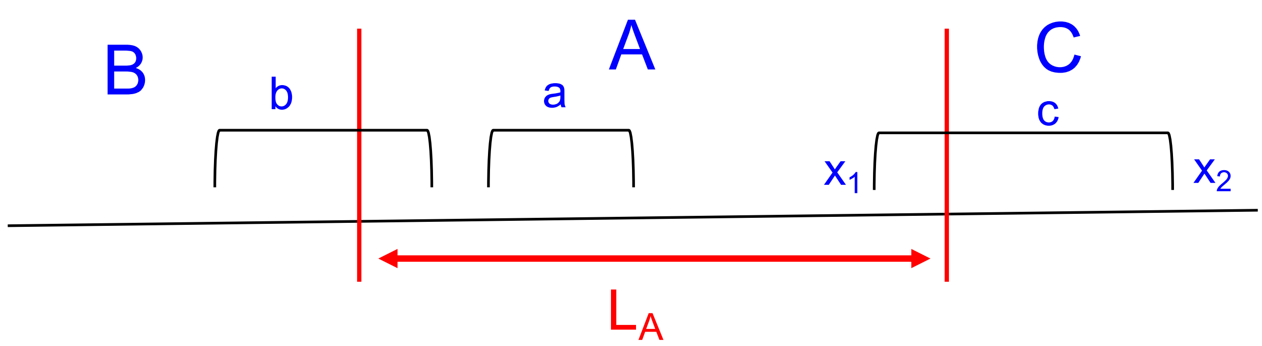
Long stabilizers + power law

Power law

Short stabilizers

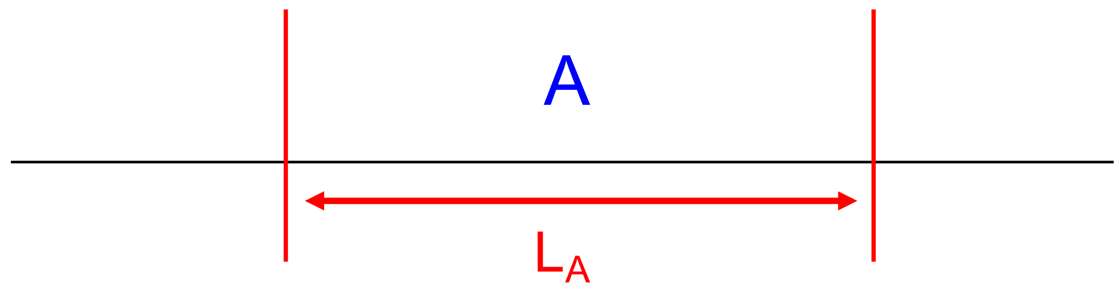
Entanglement entropy follows: $S_A = \frac{(n_b + n_c)}{2} \log(2)$

$$S_A(L_A) \approx \int_0^{L_A} dx_1 \int_{L_A}^{L} dx_2 D(x_1 - x_2, L)$$

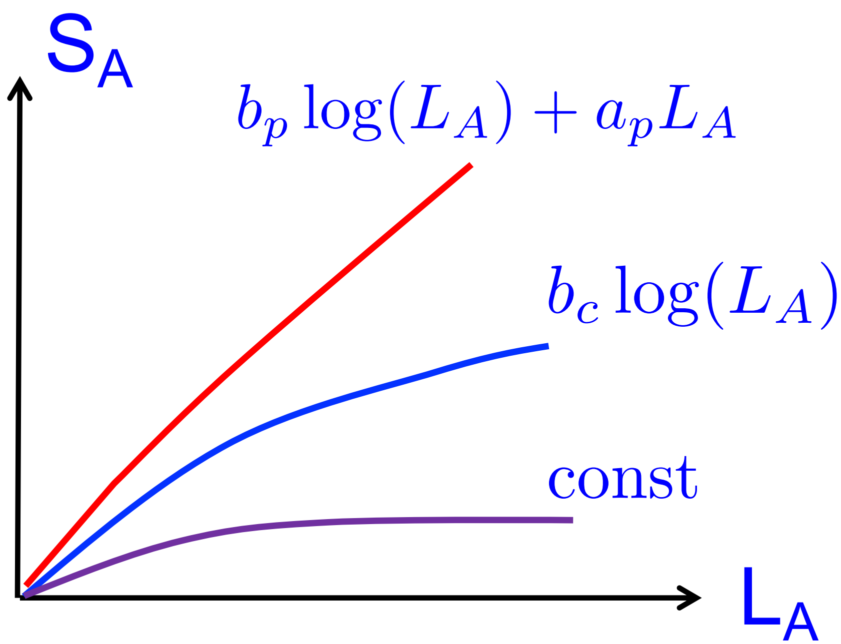


"Hidden" log inside volume-law phase

Entanglement entropy:



$$S_A(L_A) \approx \begin{cases} b_p \log(L_A) + a_p L_A; & p < p_c \\ b_c \log(L_A); & p = p_c \\ \log(\xi); & p > p_c \end{cases}$$



Beyond Clifford: Haar random Unitaries

Haar random unitaries with
single qubit projective measurements

Skinner, Ruhman, Nahum (2018)

Mapped Zeroth Renyi entropy (n=0)
to (first passage) percolation, with

$$p_c^{n=0} = 1/2$$

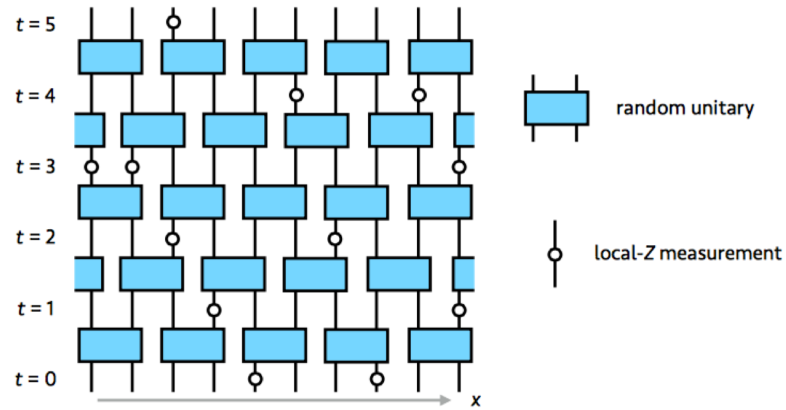
Numerics for n-th Renyi entropy;
"Different transition", $p_c < 1/2$

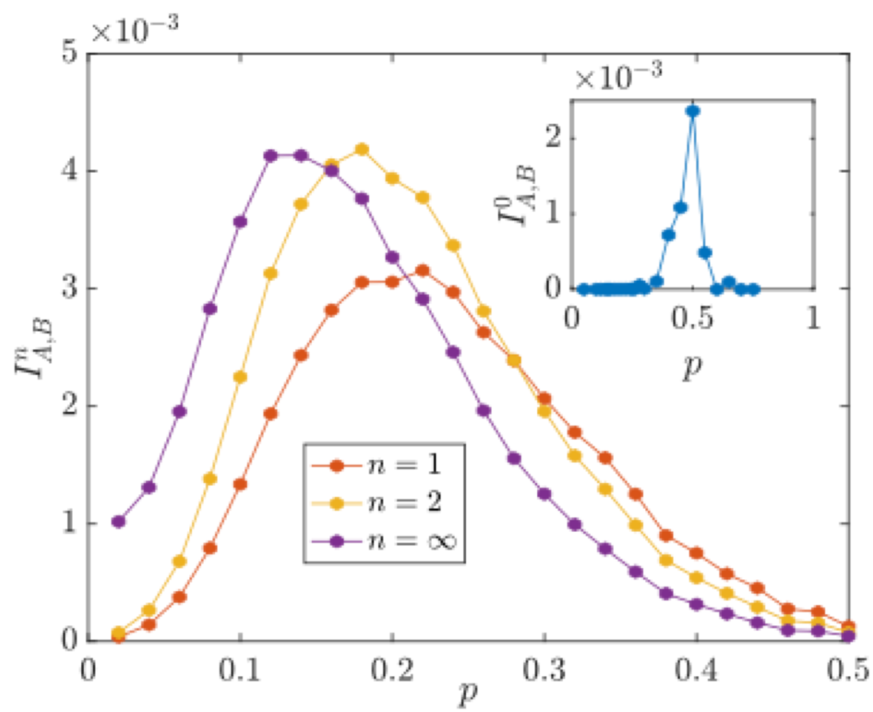
Li, Chen, MPAF (2019)

Mutual Information, varying Renyi index, n

$$I^n_{AB} = S^n_A + S^n_B - S^n_{AB}$$

$$p_c^{n \geq 1} \approx 0.2$$





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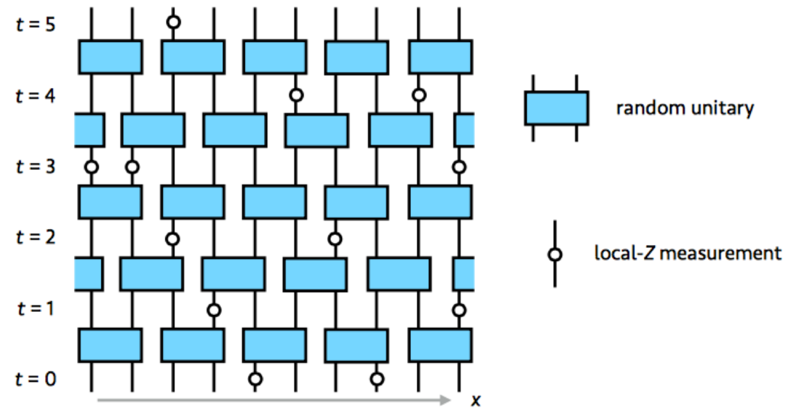
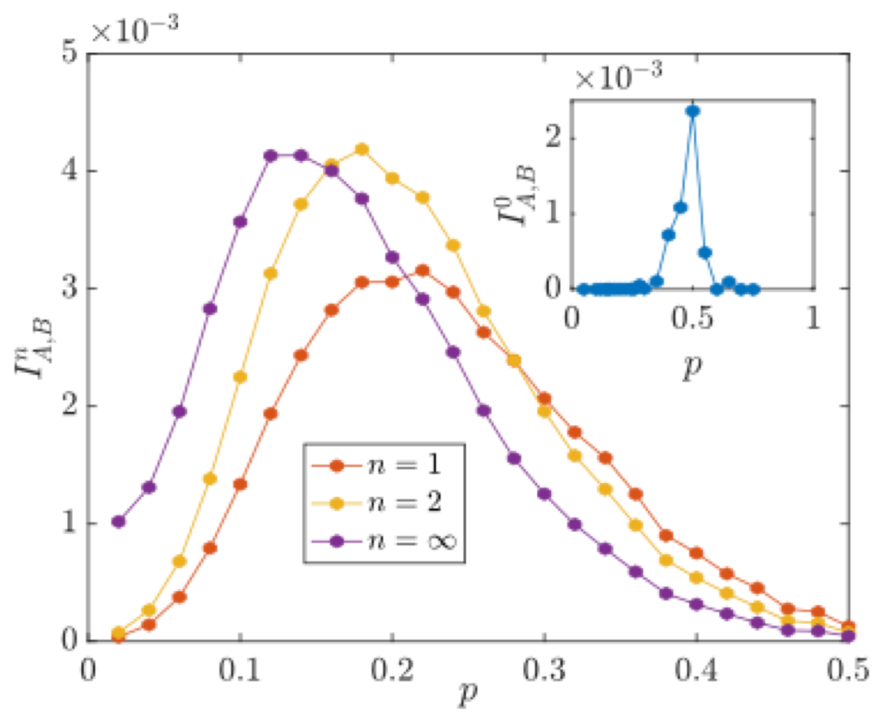
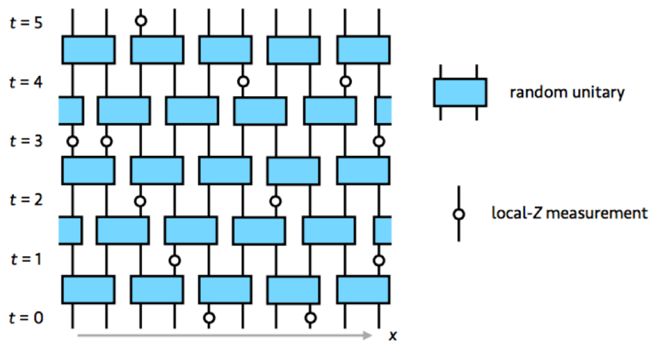




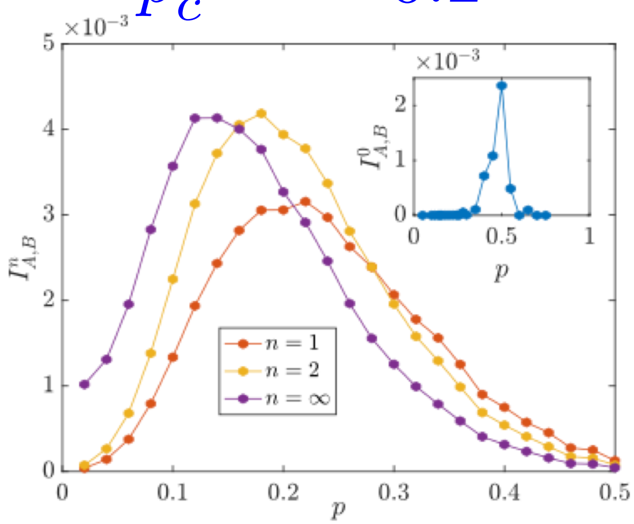
Random Haar w/ Generalized measurements

Projective measurements

$$P_\pm = \frac{1 \pm Z}{2}$$



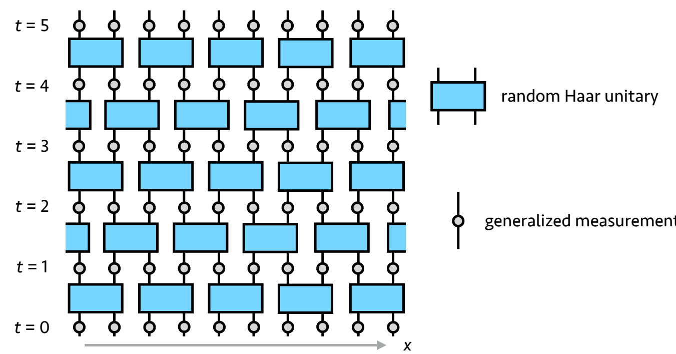
$$p_c^{n\geq 1} \approx 0.2$$



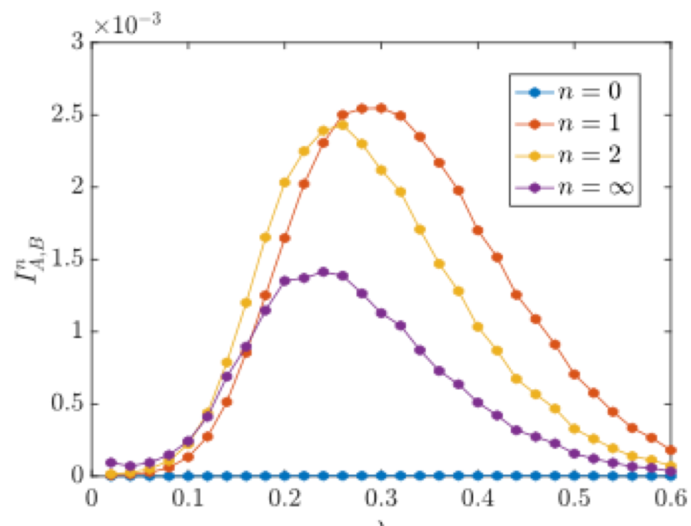
$$p_c^{n=0} = 1/2$$

Generalized measurements

$$M_\pm = \frac{1 \pm \lambda Z}{\sqrt{2(1+\lambda^2)}} \qquad \lambda \in [0,1]$$



$$\lambda_c^{n\geq 1} \approx 0.3$$



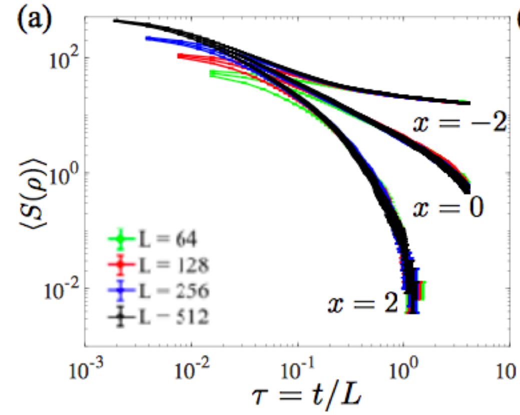
$$\lambda_c^{n=0} = 1$$

Dynamics at Purification Transition

Gullans, Huse (2019)

Dynamic scaling with z=1

$$\langle S(p,L,t)\rangle = F(L/\xi, t/L) \qquad x = (L/\xi)^{1/\nu}$$



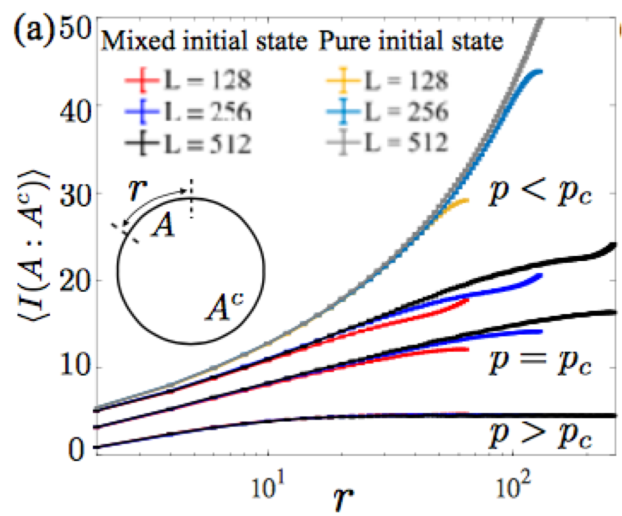
Bi-partite mutual information for $p<p_c$:

Entangled phase (steady state):

$$I_{A,\bar{A}_c} = \alpha_p \ln(L_A) + L_A/\xi$$

Mixed phase (t=4L):

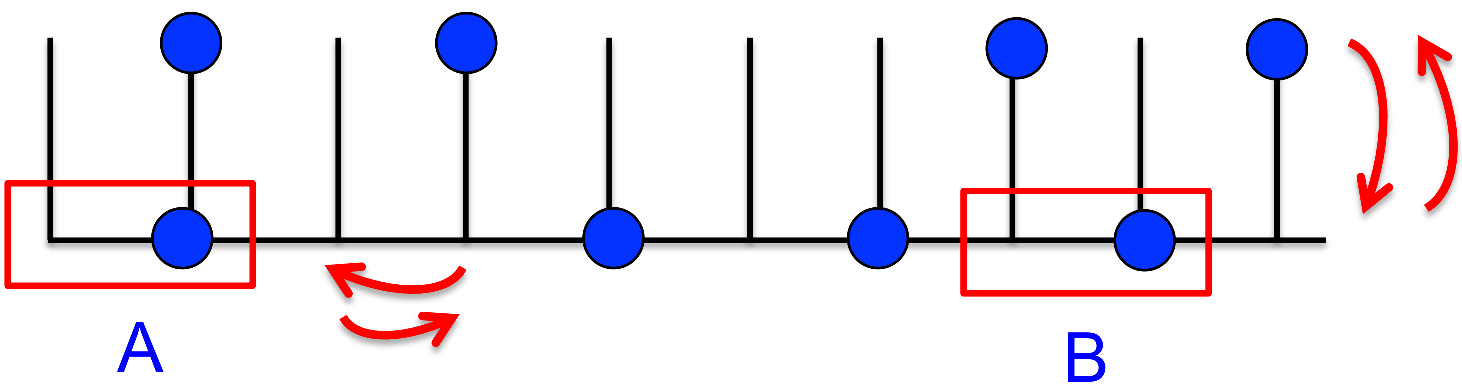
$$I_{A,\bar{A}_c} = \alpha_p \ln(L_A)$$



Log "correction" exposed in mixed phase near purification transition

Accessible in Cold Atoms/Ions?

“Comb” Lattice



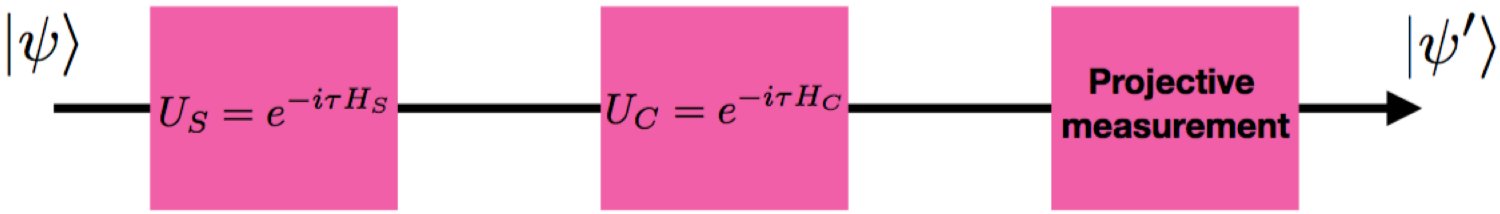
Set-up

- Bosons hopping on a “comb” lattice
- Make projective measurements on “top” of “teeth”
- Compute (and measure?) averaged-squared number fluctuation correlation function
- Expect power law decay at criticality

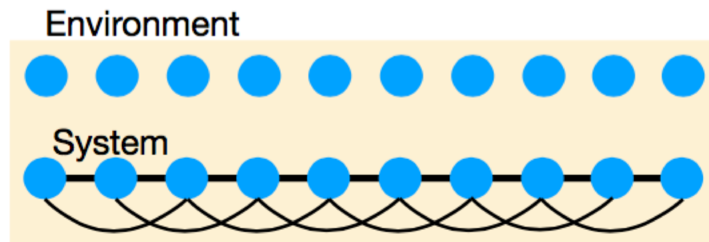
$$\overline{|\langle \delta N_A \delta N_B \rangle_c|^2} \sim |x_A - x_B|^{-\gamma}$$

Cold atoms set-up

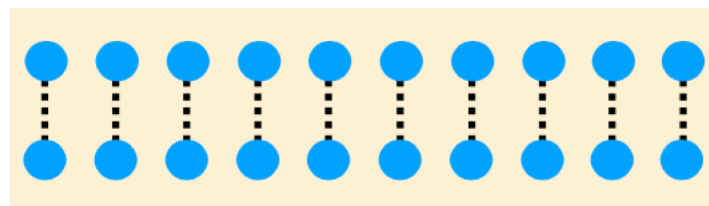
Three steps:



$$H_S = \sum_i X_i^S X_{i+1}^S + Y_i^S Y_{i+1}^S + J_1^z Z_i^S Z_{i+1}^S + J_2^z Z_i^S Z_{i+2}^S$$



$$H_C = \kappa \sum_i X_i^S X_i^E + Y_i^S Y_i^E$$



Projective measurement