Topological UQM Overview I – Fracton

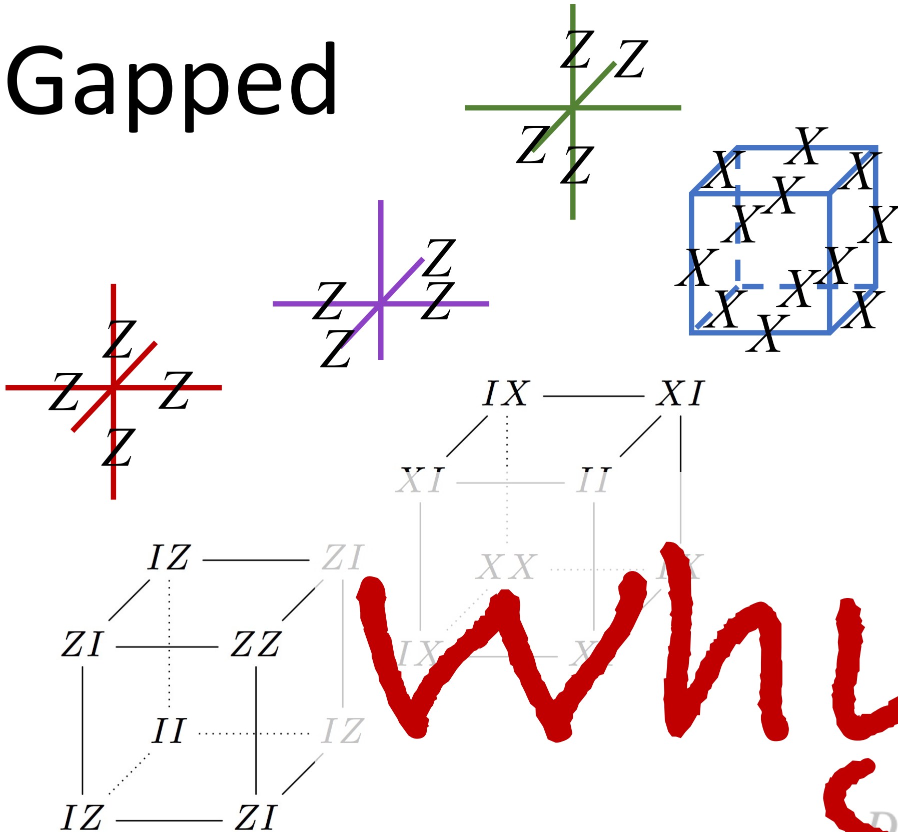
Simons Collaboration on
Ultra-Quantum Matter

Fracton

Ordinary point excitation

Fracton

Gapped



- Continuum description
- Tensor gauge theory
- Gapless modes

Why?

$$\int \rho \, d^3\mathbf{x} = 0$$

$$\int \rho\vec{x} \, d^3\mathbf{x} = 0$$

$$D_{ij}\Phi^2 = \Phi\partial_i\partial_j\Phi - \partial_i\Phi\partial_j\Phi - iA_{ij}\Phi^2$$

- Lattice model
- Exactly solvable
- Quantum Codes

$$\mathcal{L} = |D_t\Phi|^2 - m^2|\Phi|^2 - \lambda|D_{ij}\Phi^2|^2 - \lambda'\Phi^{*2}D_i^i\Phi^2 + E^{ij}E_{ij} - B^{ij}B_{ij}.$$

Gapless

Tensor gauge theory

Vector gauge theory $E_i, i = x, y, z$

Gauss's Law

$$\partial_i E_i = \rho$$

Conservation Law

$$\int \rho \, d^3\mathbf{x} = 0$$

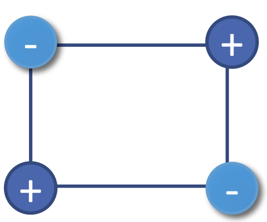
Charge Conservation

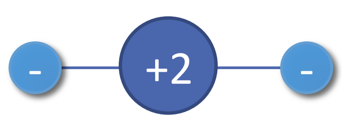
Tensor gauge theory

(Rank 2) Tensor gauge theory $E_{ij}, i, j = x, y, z$

Gauss's Law

$$\partial_i \partial_j E_{ij} = \rho$$





Conservation Law

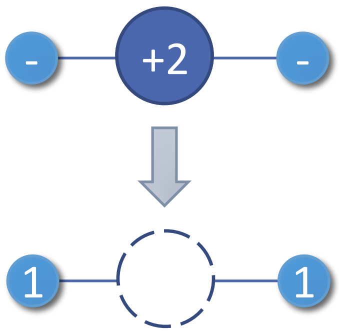
$$\int \rho \; d^3\mathbf{x} = 0$$ Charge Conservation

$$\int \rho \vec{x} \; d^3\mathbf{x} = 0$$ Dipole Conservation

Pretko, 2016

Higgsing tensor gauge theory

Higgs U(1) down to Z_2, fracton regain mobility



fracton ↓ Mobile excitation

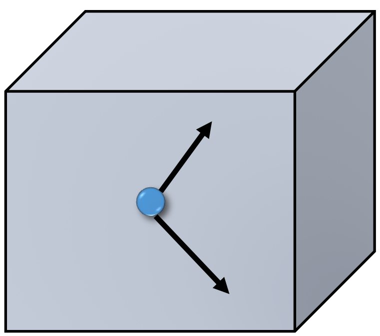
U(1) tensor gauge theory ↓ Z_2 (vector) gauge theory

Stronger constraint: subsystem symmetry

Bulmash, Barkeshli, 2018; Ma, Hermele, Chen, 2018

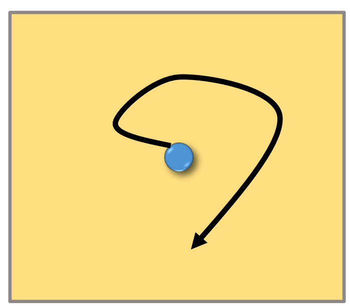
Subsystem Symmetry

Global symmetry



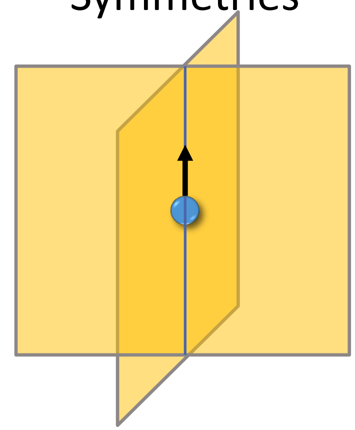
Dimension 3 Charge

Planar Symmetry



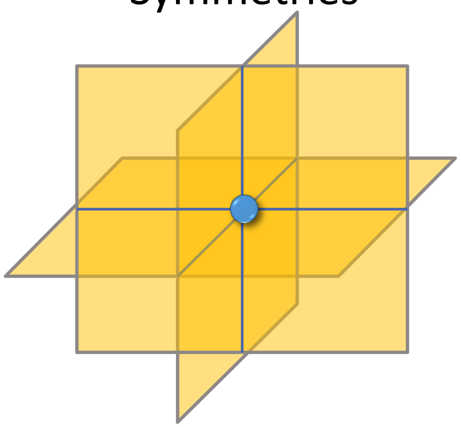
Dimension 2 Charge

2 Planar Symmetries



Dimension 1 Charge

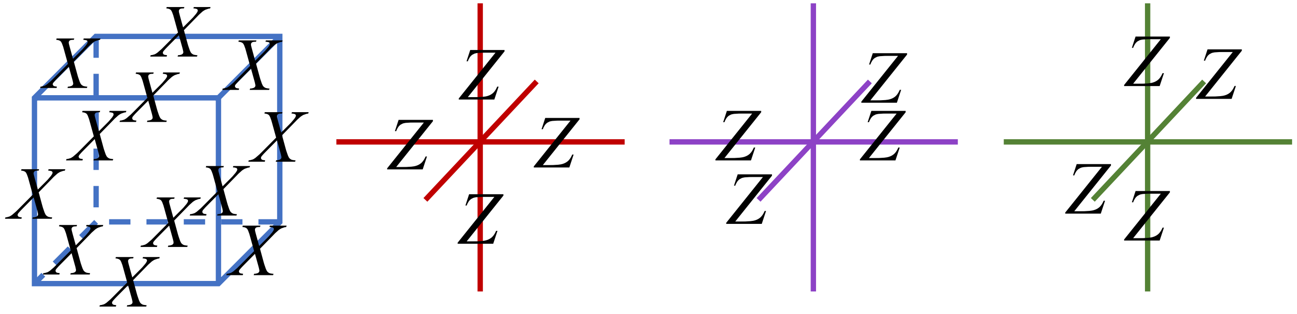
3 Planar Symmetries



Fracton Charge

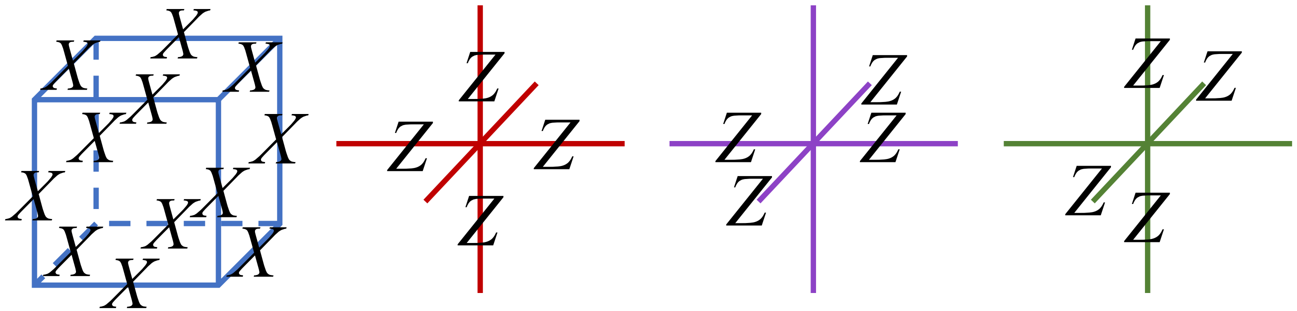
Vijay, Haah, Fu, You, Devakul, Burnell, Williamson, Sondhi, Dua, Cheng, Kubika, Yoshida…

X-cube model

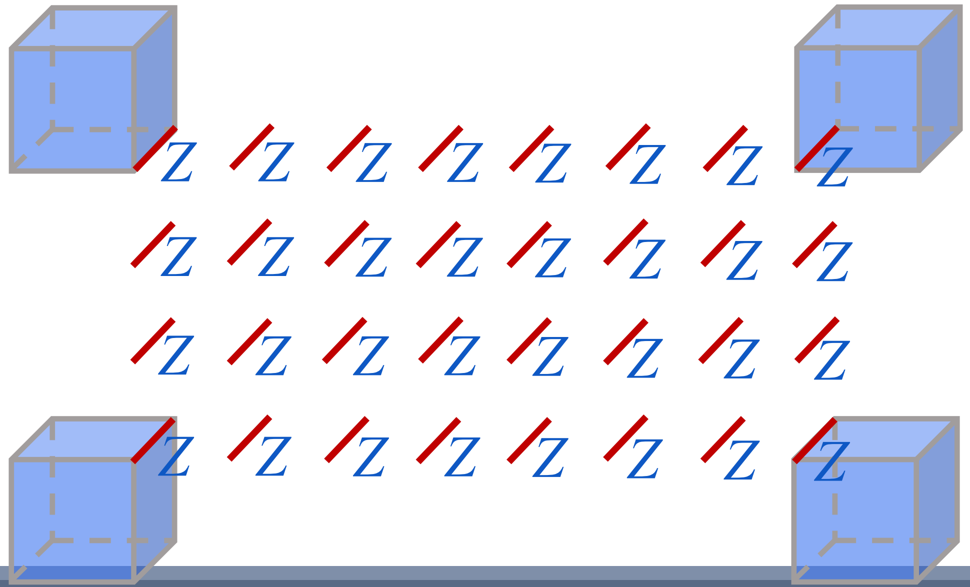


Log(Ground state degeneracy) = 6L-3

X-cube model

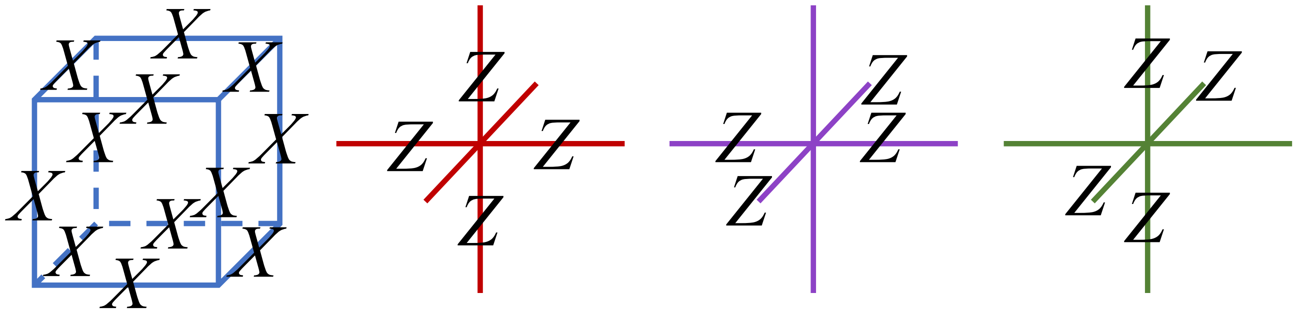


Fracton Excitations

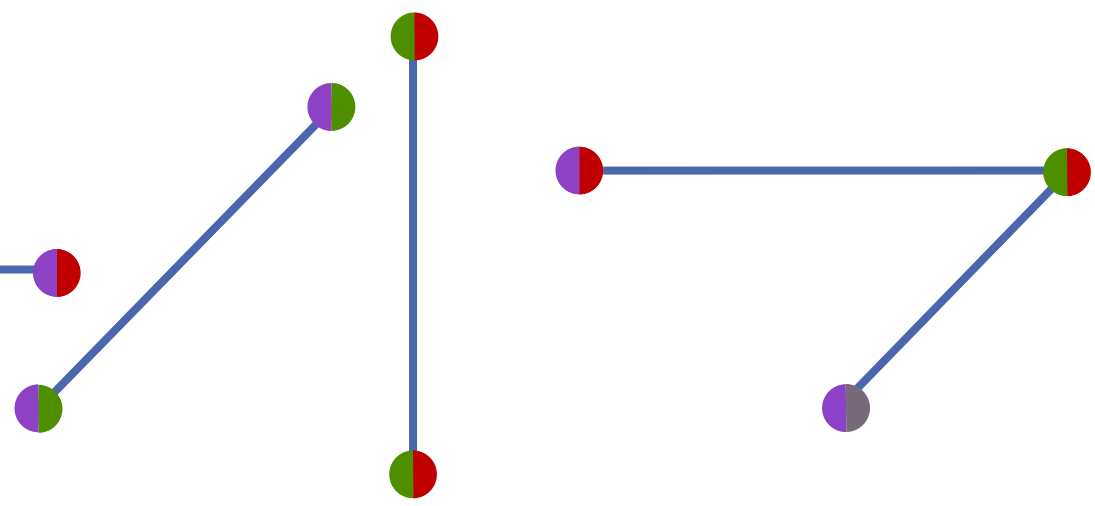


Vijay, Haah, Fu, 16

X-cube model



Lineon excitations:



Vijay, Haah, Fu, 16

Open questions and progress

- Construction of models
  - data collection
- Physical mechanism?
  - making connection with known systems
- How to characterize a fracton phase?
  - universal properties, relations between models
- Realization
  - simpler model, more realistic setup

Fracton vs. Topological Order

Similarities

- Gapped fractional excitation
- Robust ground state degeneracy on torus
- Nontrivial entanglement measures

Fracton vs. Topological Order

Relation

- Coupled layer
- Foliation
- Gauging subsystem symmetry
- Condensation, Higgs
- …

Fracton vs. Topological Order

Differences

- Excitations – restriction on mobility
- Degeneracy – extensive vs. constant
- Description in terms of field theory
- Entanglement Renormalization

Wilbur Shirley

Kevin Slagle

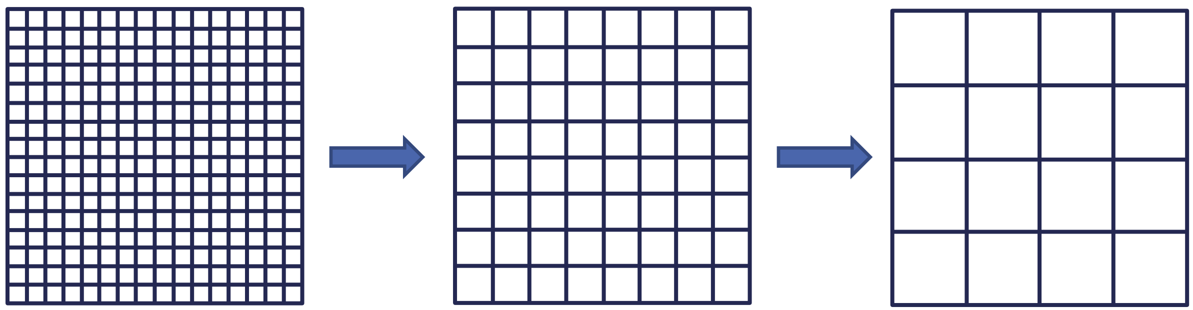
Entanglement Renormalization of Subsystem Gauge Theories

XIE CHEN, CALTECH

UQM, SEP 2019

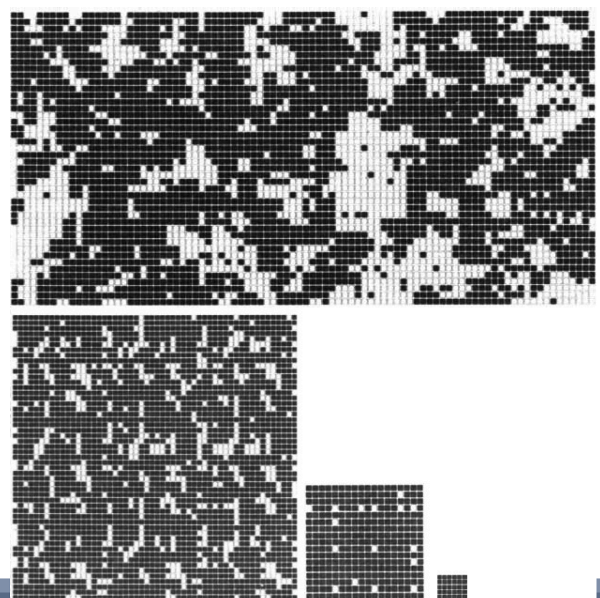
Simons Collaboration on Ultra-Quantum Matter

Real Space RG



Ising Model

$T < T_c$

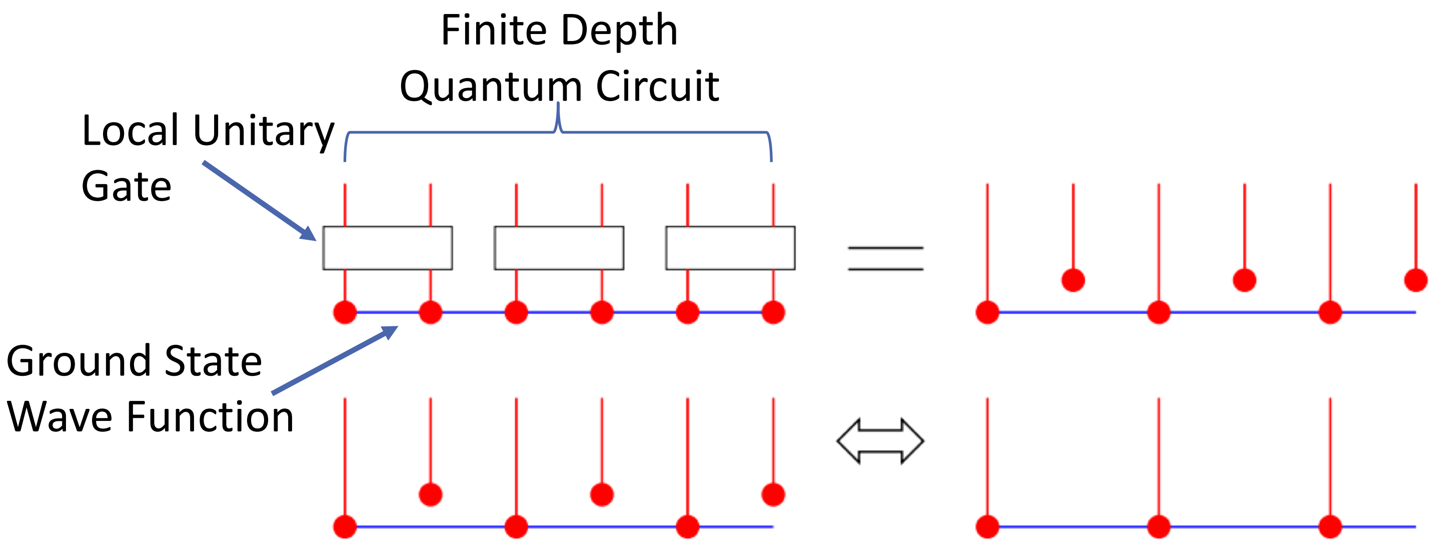


$T > T_c$

Wilson, 1979

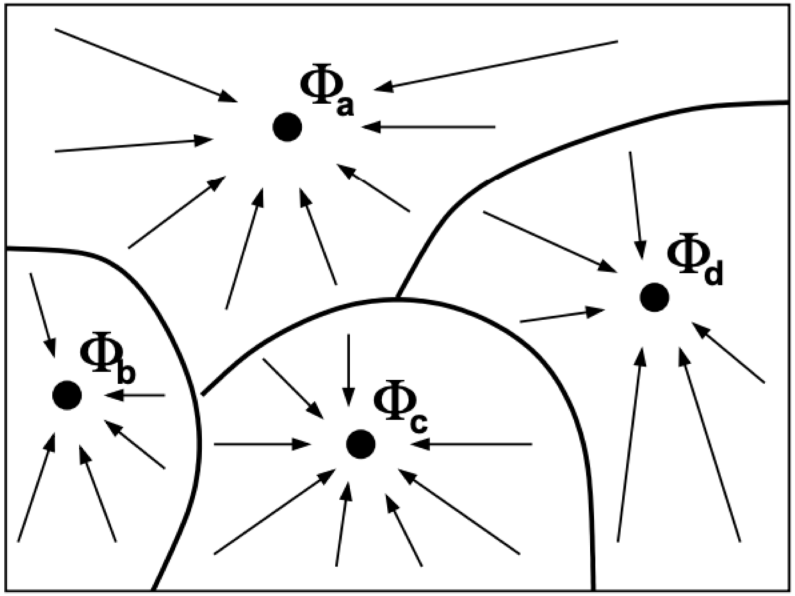
Entanglement RG

Zero T quantum phase, Gapped ground state



Entanglement RG

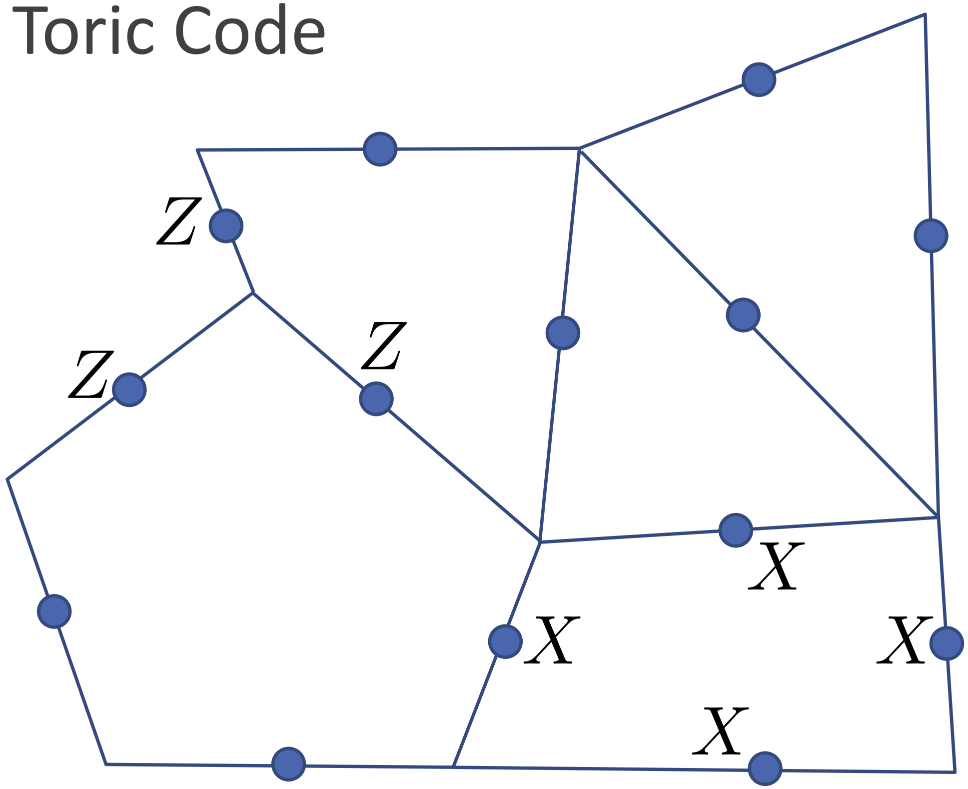
Zero T quantum phase, Gapped ground state



At fixed points, RG is reversible.

Topological Order

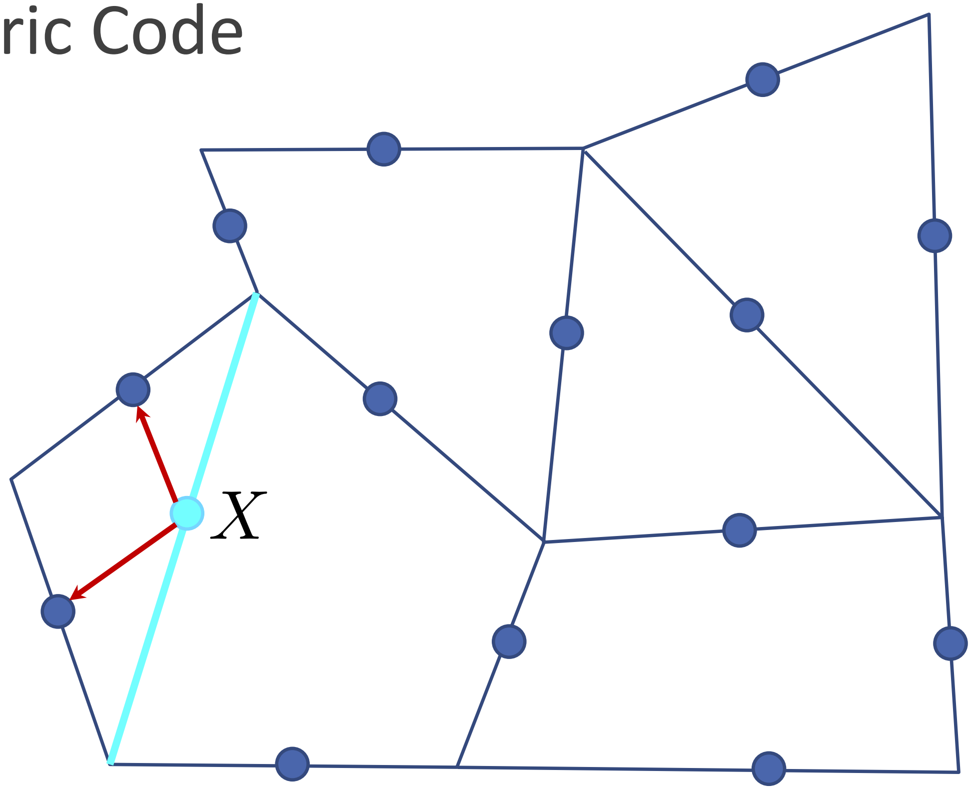
Toric Code



- 4-fold ground state degeneracy on torus
- Fractional e and m excitations
- Topological braiding statistics

Topological Order

Toric Code

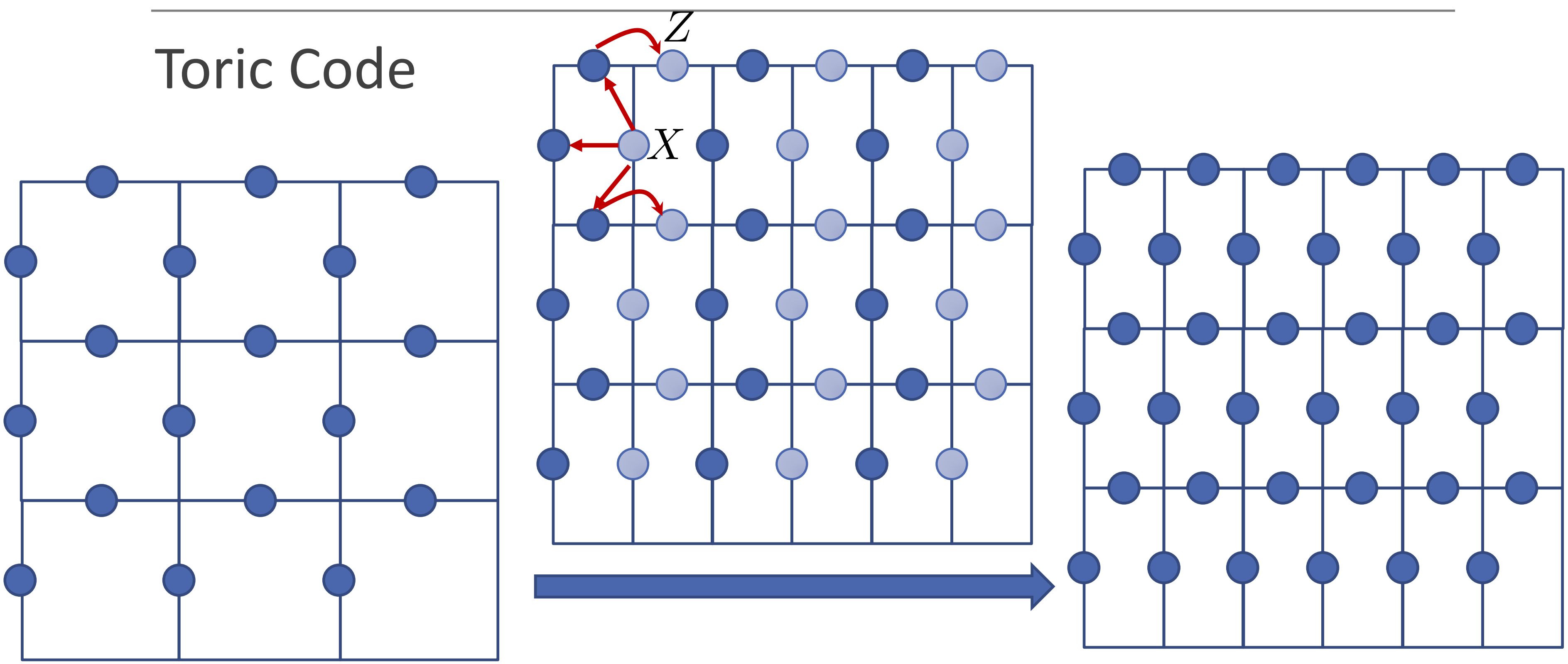


Controlled-X gate

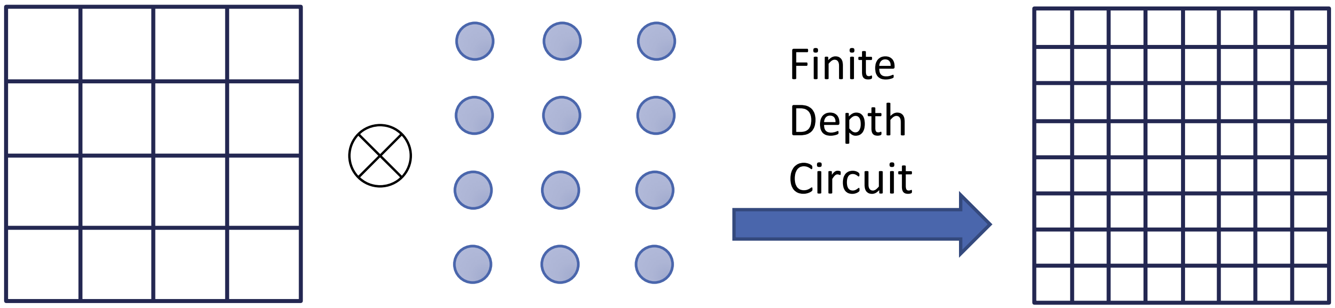
$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$$

Topological Order

Toric Code



Topological Order

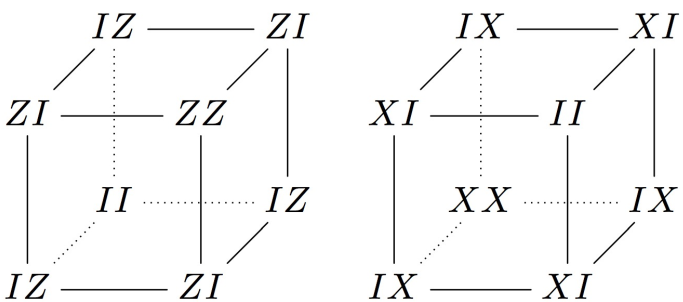


$$H(L) \otimes H_{\text{product}} \rightarrow H(2L)$$

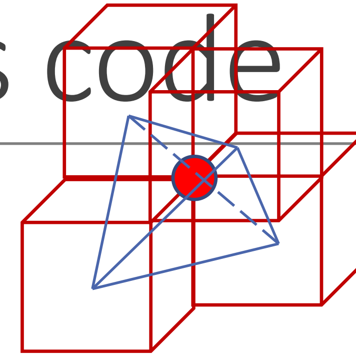
- ❑ Works for all string net, quantum doubles
- ❑ Expected to work for all topological orders
- ❑ “Liquid” topo order

Aguado, Vidal, 2008; Gu, Levin, Swingle, Wen, 2009; Konig, Reichardt, Vidal, 2009

Fracton Order: Haah's code



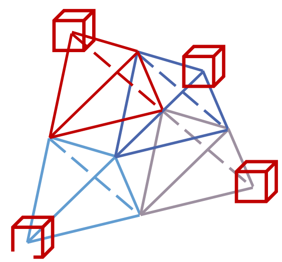
Gapped

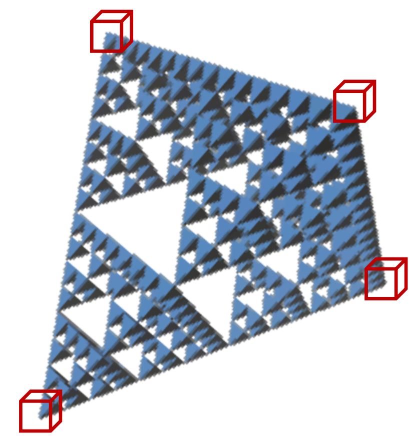


Ground State Degeneracy 2^k

$$\frac{k+2}{4} = \deg_x \gcd \begin{bmatrix} 1+(1+x)^L, \\ 1+(1+\omega x)^L, \\ 1+(1+\omega^2 x)^L \end{bmatrix}_{\mathbb{F}_4}$$

$$= \begin{cases} 1 & \text{if } L = 2^p + 1 \ (p \geq 1), \\ L & \text{if } L = 2^p \ (p \geq 1) \end{cases}$$





$$\mathbb{F}_4 = \{0, 1, \omega, \omega^2\}$$

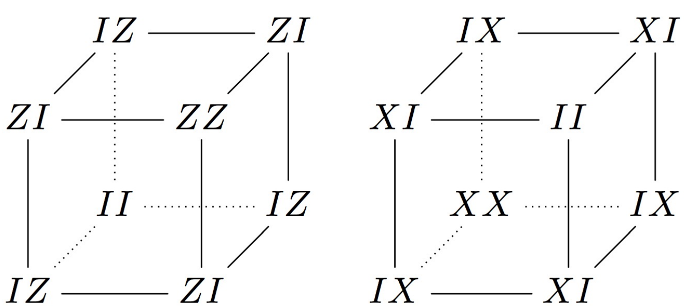
Fracton Order

Haah’s code

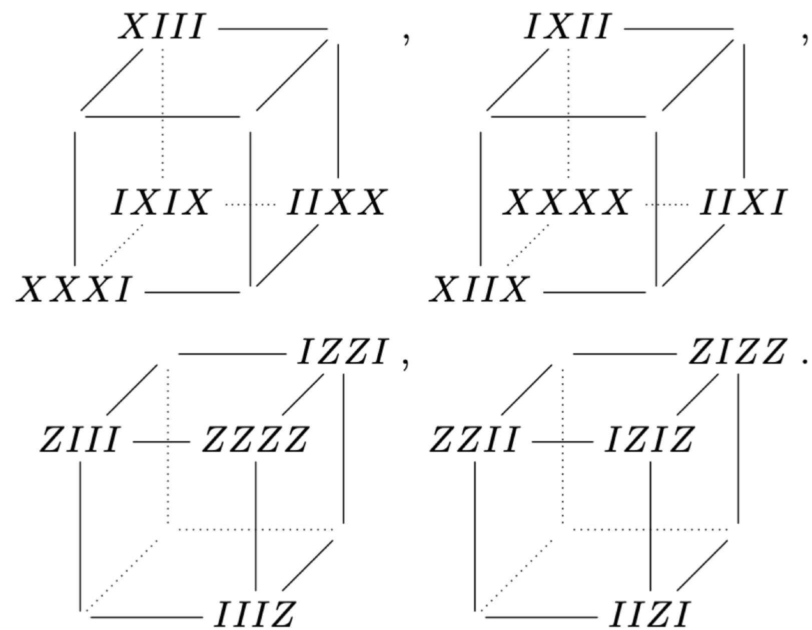
$$H_A(L) \otimes H_B(L) \to H_A(2L)$$ AB bifurcation

$$H_B(L) \otimes H_B(L) \to H_B(2L)$$ Self bifurcation

H_A



H_B



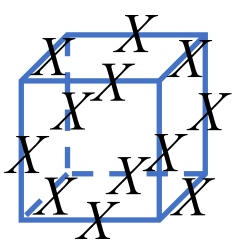
Haah, 2013

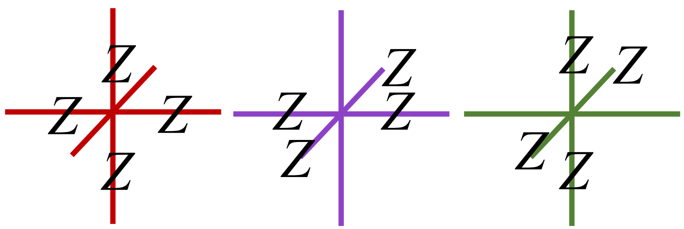
Fracton Order: X-cube model

$$H_A(L) \otimes H_B(L) \to H_A(2L)$$ AB bifurcation

$$H_B(L) \otimes H_B(L) \to H_B(2L)$$ Self bifurcation

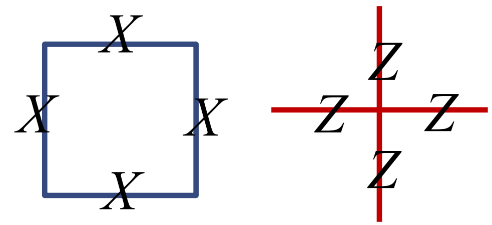
H_A

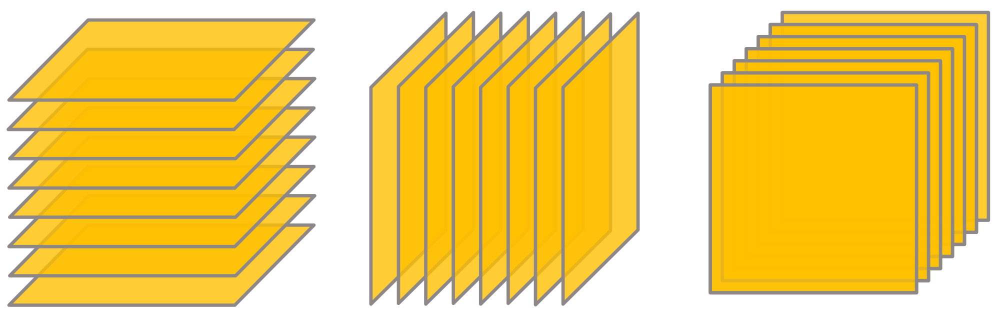




H_B

Stacks of 2D Toric Code





Shirley, Slagle, Wang, Chen, 2018

Why the difference in RG?

- When do models self-bifurcate?
- When do they AB-bifurcate?
- Other RG possibilities?

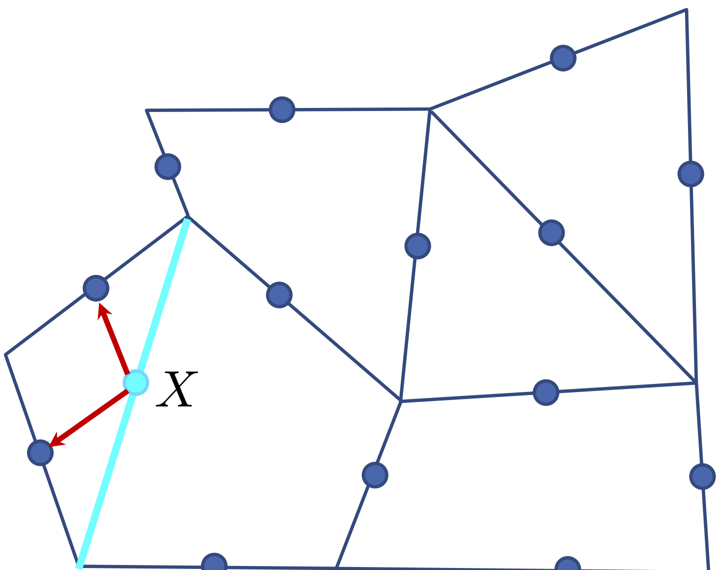
Topological / fracton order —Ungauge→ Paramagnet with global / subsystem symmetry

Paramagnet with global / subsystem symmetry —Gauge→ Topological / fracton order

Toric Code as Gauge Theory

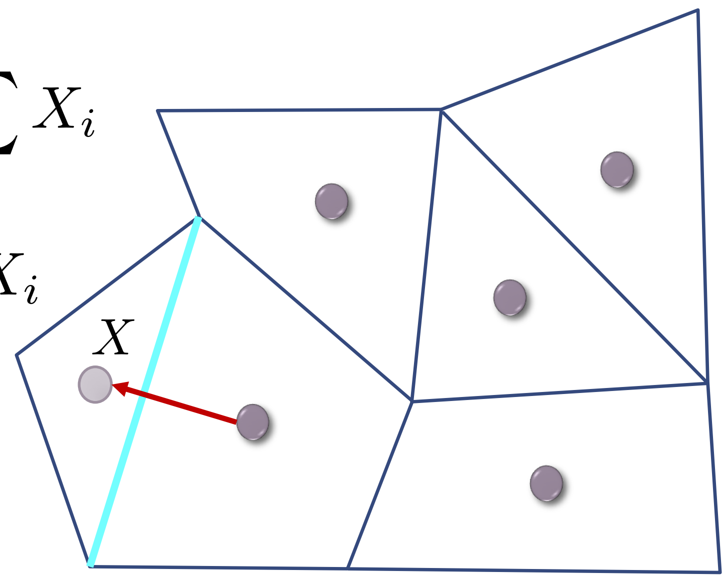
Toric Code — Ungauge → Paramagnet with global Z_2 symmetry

Z_2 gauge theory ← Gauge



$$H = -\sum_i X_i$$

$$U = \prod_i X_i$$



Toric Code as Gauge Theory

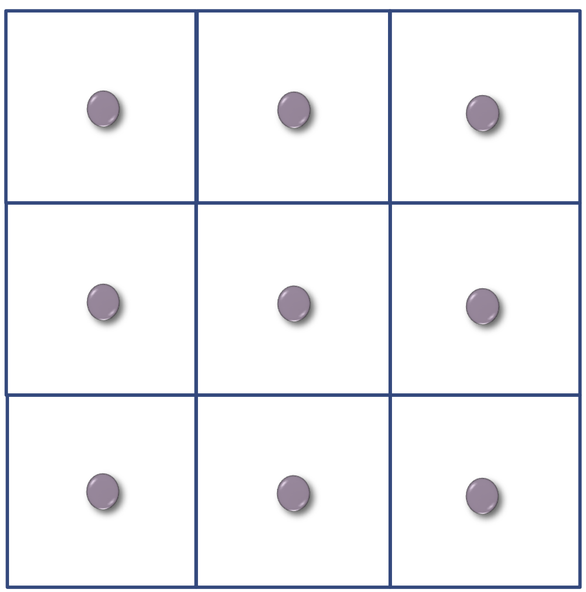
Toric Code

Z_2 gauge theory

Ungauge →

← Gauge

Paramagnet with global Z_2 symmetry



$$H = -\sum_i X_i$$

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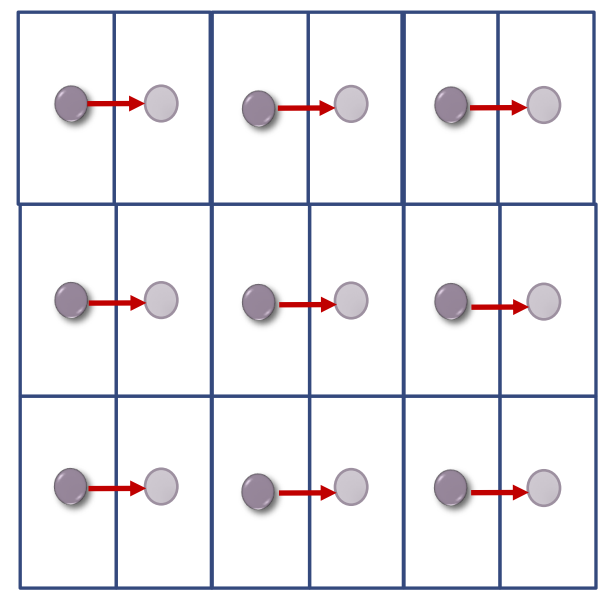
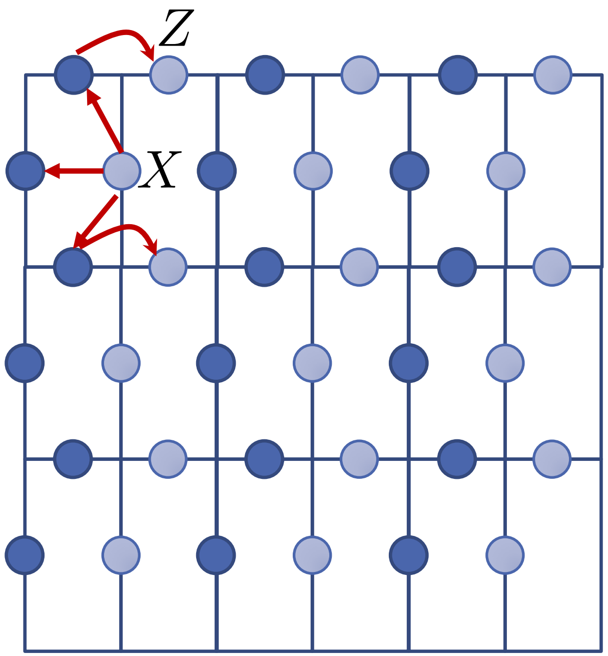


Toric Code as Gauge Theory

Toric Code — Ungauge → Paramagnet with

Z_2 gauge theory ← Gauge — global Z_2 symmetry



$$H = -\sum_i X_i$$

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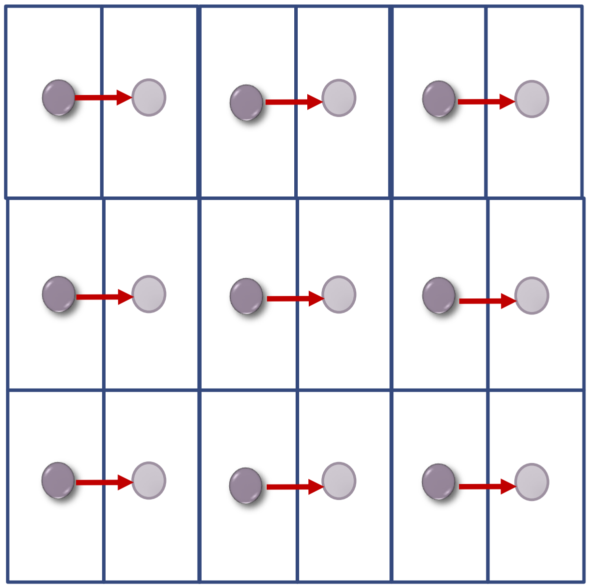
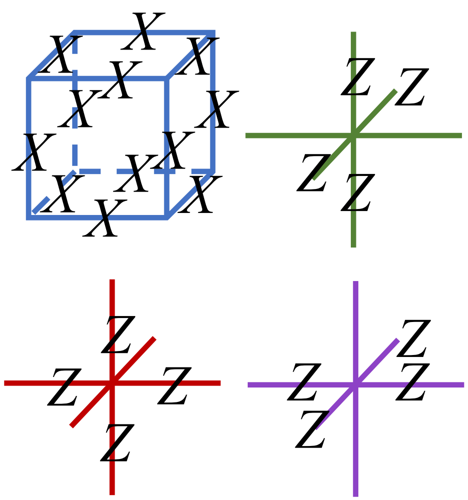
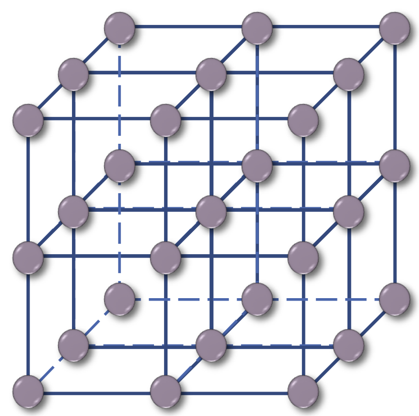


X-cube as Gauge Theory

X-cube —Ungauge→ Paramagnet with subsystem (planar) Z_2 symmetry

X-cube ←Gauge— Paramagnet with subsystem (planar) Z_2 symmetry





$$H = -\sum_i X_i$$

$$U_{\mathrm{xy}} = \prod_{i \in \mathrm{xy\ plane}} X_i$$

$$U_{\mathrm{yz}} = \prod_{i \in \mathrm{yz\ plane}} X_i$$

$$U_{\mathrm{zx}} = \prod_{i \in \mathrm{zx\ plane}} X_i$$

Vijay, Haah, Fu, 2016

X-cube as Gauge Theory

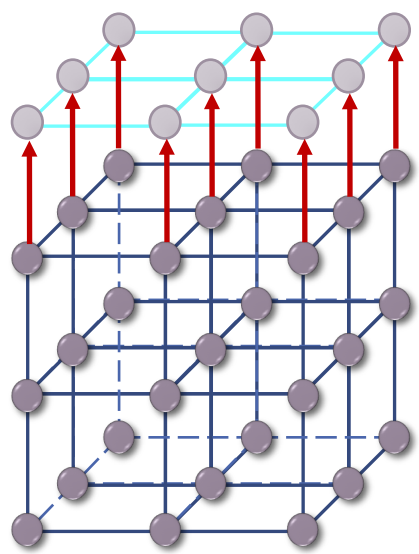
RG of paramagnet

$$H' = -\sum_i X_i$$

$$U' = \prod_{i \in \text{plane'}} X_i$$

$$U'_{\text{yz}} = \prod_{i \in \text{yz plane'}} X_i$$

$$U'_{\text{zx}} = \prod_{i \in \text{zx plane'}} X_i$$



$$H = -\sum_i X_i$$

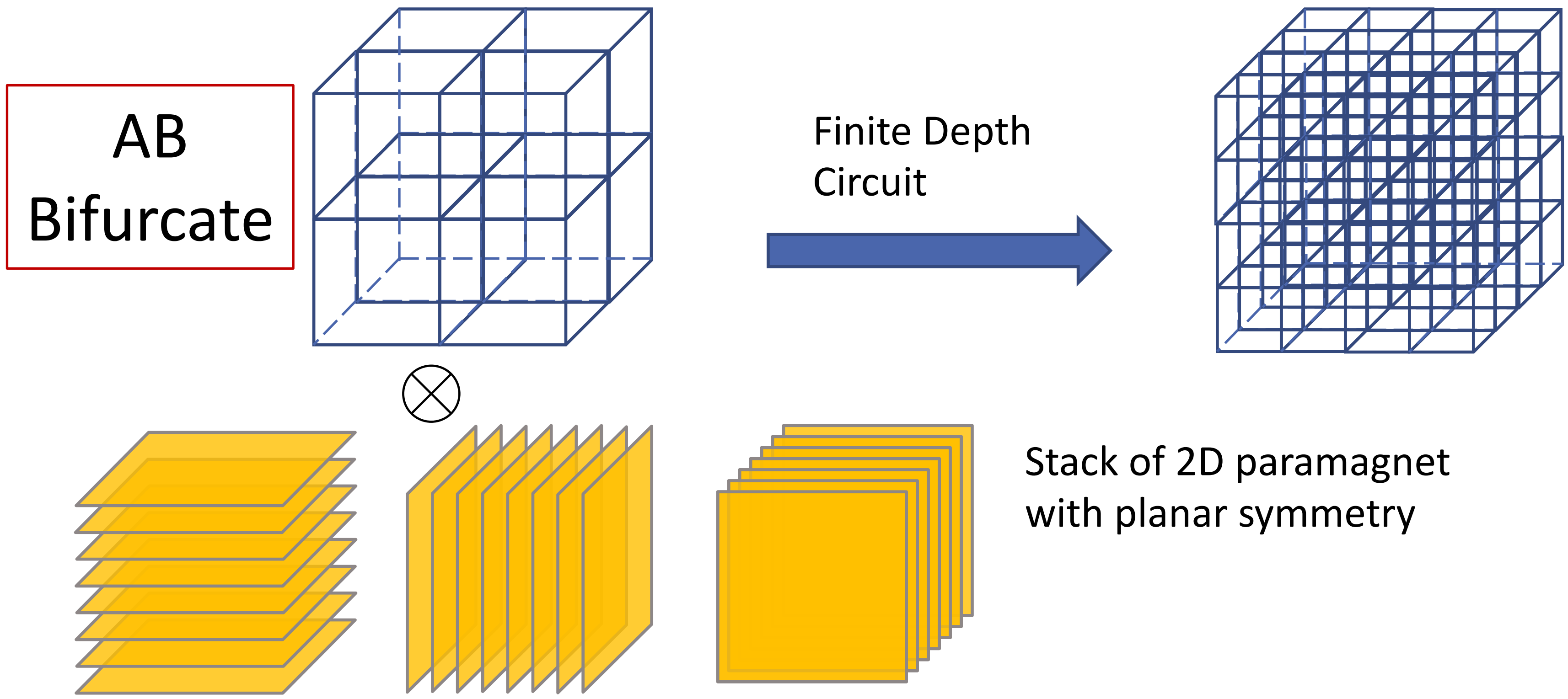
$$U_{\text{xy}} = \prod_{i \in \text{xy plane}} X_i$$

$$U_{\text{yz}} = \prod_{i \in \text{yz plane}} X_i$$

$$U_{\text{zx}} = \prod_{i \in \text{zx plane}} X_i$$

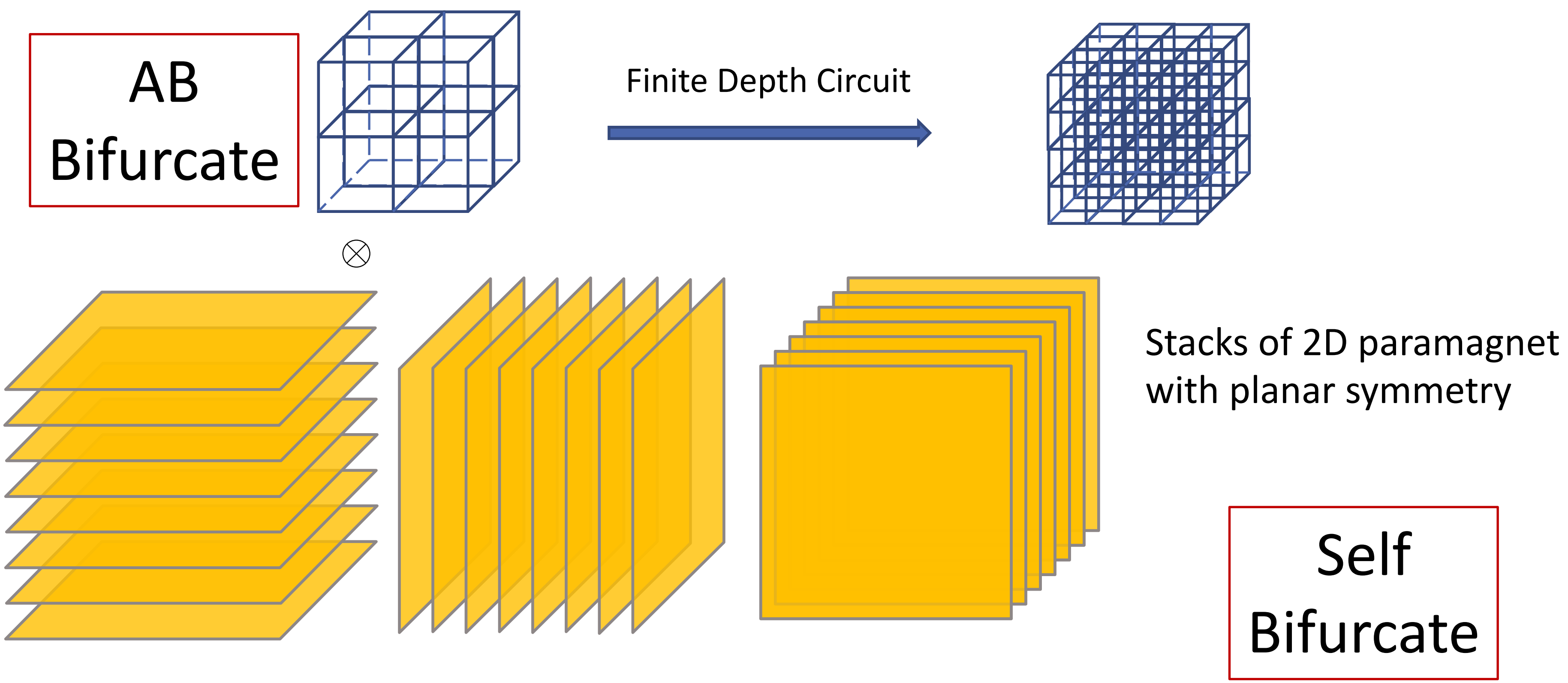
X-cube as Gauge Theory

RG of paramagnet



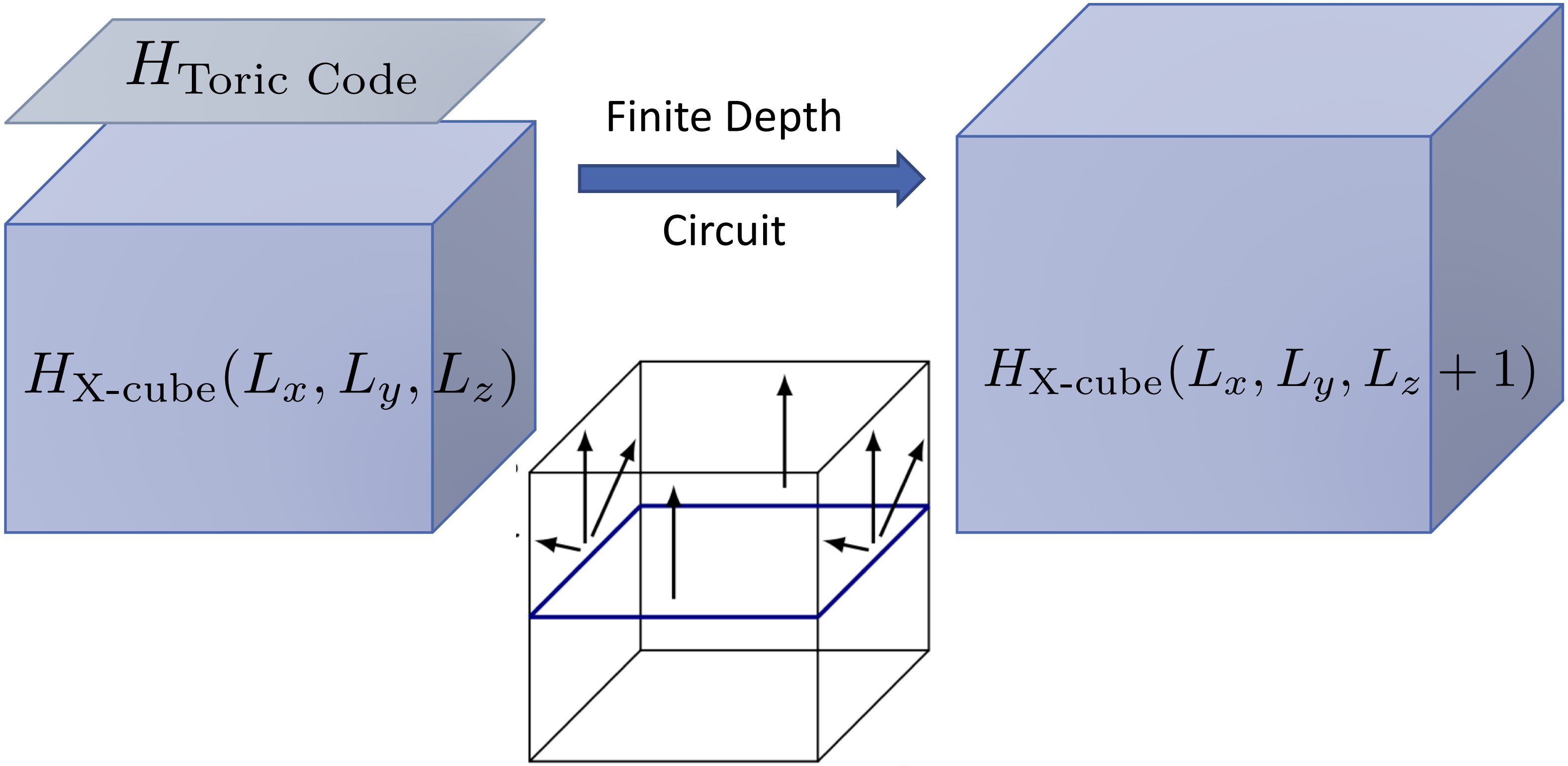
X-cube as Gauge Theory

RG of paramagnet



X-cube as Gauge Theory

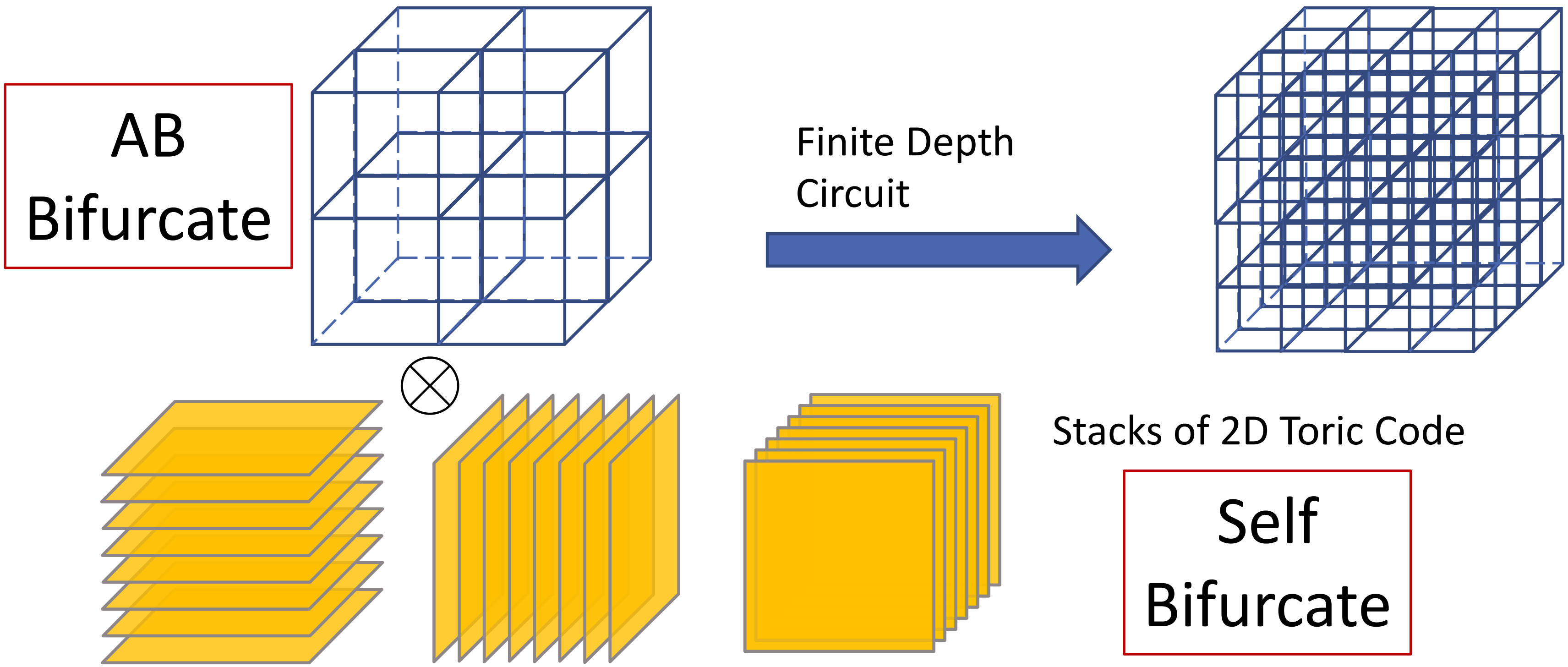
RG of X-cube



Shirley, Slagle, Wang, Chen, 2018

X-cube as Gauge Theory

RG of X-cube



What we learn from this

- Understand RG of fracton models by looking at the symmetry of the ungauged models
- When do we have ‘liquid’ topo order?
- When do fracton models self bifurcate?
- When do they AB bifurcate?
- Other RG possibilities?

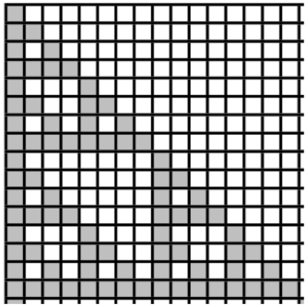
When do we have ‘liquid’ topo order?

Gauge theory of global symmetry

When do fracton models self-bifurcate?

Gauge theory of self-bifurcating symmetries:

- stacks of planar (3D) / linear (2D) sym



- sufficient condition on 2D fractal symmetries
- 3D symmetry as tensor product of 2D self-bifurcating symmetries

When do fracton models AB bifurcate?

Gauge theory of AB bifurcating symmetries:

- sufficient condition on 2D fractal symmetries
- 3D symmetry as tensor product of 2D self-bifurcating and AB-bifurcating symmetries
- Interpretation of RG transformation of Haah's code

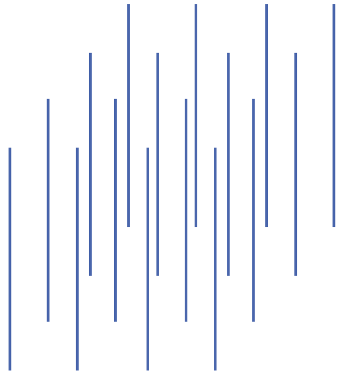
Other possibilities?

Yes. 3D paramagnet with linear symmetries in x, y, z directions.

$$H_A(L) \otimes H_B(L) = H_A(2L)$$

$$H_B(L) \otimes H_B(L) \otimes H_B(L) \otimes H_B(L) = H_B(2L)$$

H_B A 'stack' of 1D paramagnet with linear symmetry



Swingle, McGreevy, 2014

How general is this?

- 2D topological order: not everything is gauge theory
- 3D topological order: pretty much everything is gauge theory
- 3D fracton order: a large class is gauge theory; some are not clear